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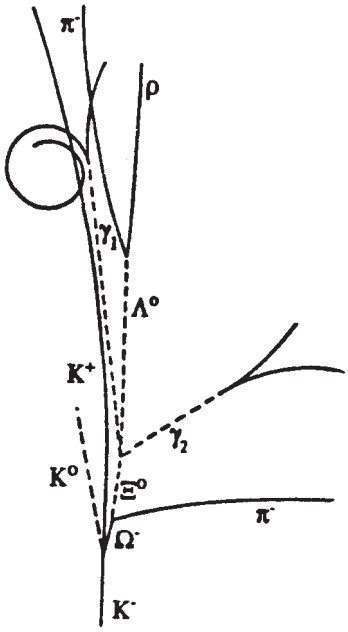

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BIG BANG (BY STERILE NEUTRINO DECAY) AND QUANTUM GRAVITY IN UNIVERSE AND CORRESPONDING ANTI-UNIVERSE

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Abstract

In this article, which is based on the works [1-31], we treat particularly the quantum gravity and the big bang. For distances smaller or equal to the Planck length, we introduce the massless universe and anti-universe, whereas for scale factors equal to or greater than the Planck length we describe the massive universe and anti-universe. By aid of a new inflation model [1-4], we derive generally the connection between vacuum energy density and cosmological "constant" for the massless and massive universes, whereat universe and anti-universe expand in opposite directions, so that we obtain a separation between matter and antimatter. On this way, for the massless universe and anti-universe, we find a simple solution for the long-sought four-dimensional quantum gravity, which via the transition from the final state of the massive universes to the big bang as the beginning of the massless universes permits the evaluation of the parameters of the big bang and the estimation of the lifetime of the sterile neutrinos, i.e. we obtain a cyclic evolution of the total (massless and massive) universes. We show that in the final state of the massive universes via the dark energy the vacuum energy density yields a gigantic energy increase in consequence of the accumulation of the decay products of the sterile neutrinos. This gigantic energy increase leads to a heat accumulation, which produces an overheat of the final state of the massive universes. This overheat means the heat death of these massive universes and simultaneously the birth of the massless universes by the hot big bang via zero-point oscillations. We explain the hypothetical superforce (of particle interactions) as well as the dark matter and the dark energy.

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1 Introduction

In this work, according to Ref. [1], we distinguish a massless ($R_{BB} \leq R \leq R_{Pl}$) as well as a massive ($R \geq R_{Pl}$) universe (anti-universe ($-R_{BB} \geq -R \geq -R_{Pl}$ as well as $-R \leq -R_{Pl}$), where R_{Pl} is the distance as well as the scale factor of the Planck length, whereas R_{BB} describes the distance of the big bang (BB). We have introduced these massless universes because their massless photons which are in thermal equilibrium with the particles, define originally their radiation energy density, so that it must be identical with their particle energy density and vacuum energy density, i.e. these massless universes contain also correspondingly massive (anti)matter. In contrast to these massless universes, the massive universe and anti-universe are characterized by identical radiation density and particle density, which however dominate the vacuum energy density, so that it can be neglected at the description of the massive universes except at the present, accelerated expansion and the final state of the massive universes. The massless universes are described by the gravitation [1], whereas the massive universes are defined by a new inflation model [1-4] and the Friedmann-Lemaitre Equations [1-11].

At $R \leq R_{Pl}$, for the massless universe (anti-universe ($-R \geq -R_{Pl}$)), we must use the quantum cosmology or quantum gravity. However, unfortunately, to this day, this quantum gravity is still highly incomplete and yields therefore no reliable predictions because the connection between cosmological "constant" and vacuum energy density is not clear [5]. Therefore, in this work, on the basis of Ref. [1], for the massless universes, by aid of the virtual matter of the quantum vacuum, we have derived a simple solution for the four-dimensional quantum gravity. Under virtual matter, we understand its formed particle-antiparticle (photon-photon) pairs, which have only an extremely short lifetime, so that they can be described however by a mean energy density and mean pressure. Because they are also formed in a radiation-free and matter-free space, the denotation "quantum vacuum" is usually applied. Taking the quantum field theory as a starting point, the total space-time continuum of the massless and the massive universe and anti-universe is penetrated with

this quantum vacuum, which forms a non-eliminative background as ground state at absence of real matter [1].

From the big bang, the momentary formation of the massless universe and anti-universe, which expand in opposite directions, leads to a separation of the virtual particle-antiparticle pairs of the quantum vacuum into real particle pairs (matter) in the massless universe and antiparticle pairs (antimatter) in the massless anti-universe via the gravitational interaction, whereat these separated particles and antiparticles are in a new thermal equilibrium with the photons, so that at the Planck length all, real particles and antiparticles have the Planck energy $kT_{\mathrm{Pl}} = E_{\mathrm{Pl}}$ as relativistic energy for their start in the correspondingly separated massive (Euclidian) universe and anti-universe [1]. Consequently, in this new thermal equilibrium, where at $R = \pm R_{\mathrm{Pl}}$ all separated particles or antiparticles have the Planck energy as relativistic energy, non-separated particles and antiparticles can be formed only by interactions between the separated particles or antiparticles, so that these non-separated particles and antiparticles disappear again by annihilation. These interactions are determined by the known laws of nuclear physics or elementary particle physics. Because the statistical particle weights were derived by aid of these laws, the new thermal equilibrium has no influence on these statistical weights [1].

Using the results of Refs. [1-4], in this work, for example, the thermal energy of the photons in the new thermal equilibrium is directly equivalent to the rest energy of the particles or antiparticles in the limiting case, where now the kinetic energy of the particles or antiparticles is zero, i.e. we can simply solve the separation of matter and antimatter [1]. In the old interpretation, where here for the universe the particles and antiparticles are produced simultaneously by photons with the thermal energy of the sum of their rest energy in the thermal equilibrium, so that this old interpretation has difficulties at the explanation of the separation of matter and antimatter [1].

We have shown in Ref. [1] that from the big bang the immediately formed, massless universe and anti-universe expand in opposite direction, so that we obtain directly and correspondingly the separation of the virtual matter [photon(particle)-photon(anti-particle) pairs] of the massless universe and

antimatter [photon(anti-particle)-photon(particle) pairs] of the massless anti-universe into real matter [photon(particle)-photon(particle) pairs] and antimatter [photon(antiparticle)-photon(antiparticle) pairs] as a real radiation field and a real particle field ($R_{BB} \leq R \leq R_{Pl}$) as well as a real radiation field and an antiparticle field ($-R_{Pl} \leq -R \leq -R_{BB}$) in the massless universe and anti-universe, respectively.

For the massless universes, which must result from zero-point oscillations, we have so derived a simple solution for the long-sought, four-dimensional quantum gravity, which via the transition from the final state of these massive universes to the big bang permits the determination of the parameters of the big bang and the evaluation of the lifetime of the sterile neutrinos [1]. Thus, we obtain a cyclic evolution of the total (massless and massive) universes as a result of the dark energy converted half into massless photons or massive (neutrino) relics by the decay of the sterile neutrinos via the gravitation [1]. This decay process must lead to an overheat of the final state of the massive universes, i.e. it must take place their heat death and simultaneously the birth of the massless universes as big bang via zero-point oscillations [1].

In works [1, 2], we have derived the vacuum energy density for $R = \pm R_{Pl}$ via the new inflation model [1-4], defined by the supersymmetric grand unification particles (X and Y gauge bosons [1-4] as well as magnetic monopoles [1-4]) in the massive universe and anti-universe. This derivation of the vacuum energy density is used as fundamental magnitude for this work, based on Ref. [1], whereat for $R = \pm R_{Pl}$ we find the same value by a new expression for this vacuum energy density via a new, modified, quantum-statistical photon energy density. For example, this new expression of the vacuum energy density can be generalized for the massive universes ($R \geq R_{Pl}$ and $-R \leq -R_{Pl}$). Using this new expression from $R = \pm R_{Pl}$ for the massless universes ($R_{BB} \leq R \leq R_{Pl}$ and $-R_{Pl} \leq -R \leq -R_{BB}$), we find also a general expression of their vacuum energy density as quantum gravity, derived by aid of the gravitation [1]. The works [1-4] show clearly that the massive universe is completely described via the new inflation model [1-4] and the Friedmann-Lemaitre Equations [1-11] for Euclidian geometry [1-4] by the (present),

cosmological parameter values [1-11], which were also exactly estimated in Refs. [1-3] via the light neutrino density parameters [1-4], multiplied by the different ratios of the relativistic energy and the rest energy of the supersymmetric grand unification particles [1-3]. On this way, because of the known, general properties of particles and antiparticles, for the massive anti-universe ($R \geq R_{\mathrm{Pl}}$), in work [1], we have derived also the (present), cosmological parameter values similar as at the massive universe, using the antiparticles of the light neutrinos and the supersymmetric grand unification particles. Thus, we assume that the massive anti-universe is also described completely as the massive universe [1-4], determining the necessary quantities of the Friedmann-Lemaitre Equations for the massive anti-universe on the basis of the general properties of particles and antiparticles [1].

In this work, we extend the results of the works 1-4].

Thus, this work is organized as follows. In Sec. 2, we describe shortly the Friedmann-Lemaitre Equations as well as their most important, known solutions, whereat we include SUSY GUTs for the early universe. In Sec. 3, we describe the properties of the light neutrinos (Sec. 3.1) and the SUSY GUT particles including the new inflation model (Sec. 3.2), whereat we treat the rest energy of the light neutrinos (Sec. 3.1.1), the particle density parameters of the light neutrinos (Sec. 3.1.2), the necessary properties of the X and Y gauge bosons (Sec. 3.2.1), the "two scale factors" of absorption of light at the redshift condition (Sec. 3.2.2), the necessary properties of the magnetic monopoles (Sec. 3.2.3), the new inflation model for the massive universe and anti-universe (Sec. 3.2.4) as well as the determination of the vacuum energy density by the new inflation model via quantities of the old inflation model at $R = R_{\mathrm{Pl}}$ (Sec. 3.2.5). In Sec. 4, we treat the heavy neutrinos, whereat we describe the rest energy and number density of the heavy neutrinos (Sec. 4.1) and the determination of the particle-defined (present) cosmological parameter values via the rest energy of the heavy neutrinos (Sec. 4.2). In Sec. 5, we describe the sterile neutrinos, whereat we treat the rest energy and the number density of the sterile neutrinos (Sec. 5.1), the derivation of the particle-defined, (present) cosmological parameter values via the rest energy of the sterile neutrinos (Sec. 5.2) as well as a semi-empirical explanation for

the sterile neutrino calculation (Sec. 5.3). In Sec. 6, we derive a simple solution for the four-dimensional quantum gravity via the vacuum energy density, whereat we treat the vacuum energy density at $R = R_{\mathrm{Pl}}$ as a consequence of a modified, quantum-statistical expression for the photon energy density (Sec. 6.1), the vacuum energy density directly via the gravitation by photons and particles at $R = R_{\mathrm{Pl}}$ (Sec. 6.2), the quantum gravity as a result of the vacuum energy density via the gravitation by photons and particles for $R \leq R_{\mathrm{Pl}}$ (Sec. 6.3), the vacuum energy density at $R \geq R_{\mathrm{Pl}}$ as quantum gravity via a modified, quantum-statistical expression for the photon energy density (Sec. 6.4) as well as several conclusions from the vacuum energy density (Sec. 6.5). In Sec. 7, we describe new results for the universe and the anti-universe as the cosmic, final state and the end of the present, exponential expansion by a negative acceleration (Sec. 7.1), the astronomical unit changing (Sec. 7.2), the cosmic future by a slow, linear expansion after the present, accelerated expansion (Sec. 7.3) as well as the redshift values of the reionization (Sec. 7.4). In Sec. 8, we calculate the transition from the final state of the massive universe to the big bang, whereat we treat the parameters of the big bang (Sec. 8.1) the lifetime of the sterile neutrinos (Sec. 8.2), the constant volume of the sterile neutrinos (Sec. 8.3), the zero-point oscillations as the origin of the massless universes (Sec. 8.4), the particle horizon (Sec. 8.5) and the hypothetical superforce of particle interactions (Sec. 8.6). In Sec. 9, we explain the present, dark matter and dark energy. In Sec. 10, we give a short summary. The values of the physical constants, used in this work, are given in works [6, 9].

2 The Friedmann-Lemaitre Equations (with the most important, known solutions of the standard model) as well as SUSY GUTs

In this chapter, the considerations of the Friedmann-Lemaitre Equations and their most important, known solutions are based on Refs. [1-11].

In agreement with all of those parts of the universe, which are accessible by observations, we can assume that at (very) early times and today on a

sufficiently large scale the universe is very nearly homogeneous and isotropic. For such a space, the most general four-dimensional line element ds is determined by the Friedmann-Robertson-Walker metric in the form

$$ds^2 = c^2 dt^2 - R^2(t)\left[\frac{dr^2}{1-\mathrm{k}r^2} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right], \tag{2.1}$$

where c is the velocity of light in vacuum and t is the time, whereas the dimensionless Lagrange coordinates r, ϑ and φ are constant because of the expansion of the universe, so that $R(t)$ determines alone the time dependence of this metric, i.e. $R = R(t)$ is the scale factor for distances in comoving coordinates. By appropriate rescaling of the radial coordinate, the constant k can be chosen to be $+1$ (closed geometry with finite volume), -1 (open hyperbolic space) or 0 (Euclidian (spatially flat) geometry with infinite volume). In such an expanding universe, the observed wavelength λ of the light emitted (e) from a distant source with λ_e, is shifted towards the red. Via the Friedmann-Robertson-Walker metric, this so-called redshift z is defined by

$$1 + z = \frac{\lambda}{\lambda_e} = \frac{R_0}{R}, \tag{2.2}$$

where R_0 and R are the scale factors of absorption and emission of light, respectively. In the Friedmann-Robertson-Walker metric, Einstein's equations lead to the Friedmann equation

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = -\mathrm{k}\frac{c^2}{R^2} + \frac{8\pi G_N}{3}\rho + \frac{c^2}{3}\Lambda \tag{2.3}$$

as well as to the equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda \tag{2.4}$$

as the so-called Friedmann-Lemaitre Equations, where $H = H(t)$ is the Hubble parameter, G_N is the gravitational constant, ρ is the mean mass density, P is the isotropic pressure and Λ is the cosmological constant.

In Refs. [1-4], for Eq. (2.3), we have generally derived $k=0$ by $\Omega_{tot}=1$ and $\Omega_{tot}(z)=1$, i.e. in this work the Friedmann equation (2.3) can be assumed to

$$H^2=\left(\frac{\dot{R}}{R}\right)^2=\frac{8\pi G_N}{3}\rho+\frac{c^2}{3}\Lambda\,. \tag{2.5}$$

Additional to the Hubble parameter, it is usual to define also several, other, measurable cosmological parameters. Their most important parameter values are summarized in Table I, which contains the critically analyzed, observed, (present), cosmological parameter values for the massive universe according to Ref. [6]. For better comparison of observed and estimated cosmological parameter values, we mention here also their measured Planck 2013 data [11], summarized in Table II.

In Ref. [1], we have shown that for the description of the massive anti-universe, the (present), cosmological parameter values of Tables I and II are also valid, except the values H_0 and h, which are here negative. The reasons of this behavior are the known properties of particle and antiparticle according to Table III.

Using these special properties of particles and antiparticles in Table III, we have also determined the most important quantities, which are necessary for the description of the massless and massive universes [1]. These quantities are summarized in Table IV.

In Refs. [1-3], we have proved that these cosmological parameters of Tables I and II are excellently described by particle-defined, (present), cosmological parameter values, calculated by the light neutrino density parameters as well as the ratios of the relativistic energy to the rest energy of the SUSY GUT particles. Thus, in Ref. [1], by aid of the data of Tables III and IV, we have shown that these particle-defined, (present), cosmological parameters are also valid for the anti-universe. We summarize these particle-defined, (present), cosmological parameters in Table V. Because of the excellent agreement between these observed (Tables I and II) and calculated (Table V), cosmological parameters, we use always the cosmological parameter values of Table I in this work for calculations.

TABLE I. The most important, critically analyzed, (present), cosmological parameter values [6] of the massive universe for the Standard model. The abbreviation for cosmic microwave background is CMB. [a)] Calculated by $h = 0.673 \pm 0.012$. [b)] Different fits [6]

Quantity	Symbol, equation	Value
present day CMB temperature	T_0	$2.7255(6)\,\mathrm{K}$
present day Hubble expansion rate	H_0	$67.3 \pm 1.2\,\mathrm{km\,s^{-1}\,Mpc^{-1}} =$ $= h \times (9.777752\,\mathrm{Gyr})^{-1} =$ $= (2.181 \pm 0.039) \cdot 10^{-18}\,\mathrm{s^{-1}}$
scale factor for Hubble expansion rate	h	0.673 ± 0.012
critical present density of the universe	$\rho_{0C} = 3H_0^2 / 8\pi G_N$	$1.05375(13) \cdot 10^{-5} h^2 (\mathrm{GeV}/c^2)\,\mathrm{cm^{-3}} =$ $= 4.77(17) \cdot 10^3 (\mathrm{eV}/c^2)\,\mathrm{cm^{-3}}$
baryon (proton) density of the universe	Ω_b	$0.02207(27)\,h^{-2} = 0.0487(23)$ [a)], $0.0499(22)$ [b)]
(cold) dark matter density of the universe	Ω_{dm}	$0.1198(26)\,h^{-2} = 0.265(15)$ [a)], $0.265(11)$ [b)]
dark energy density of the universe	Ω_Λ	$0.685^{+0.017}_{-0.016}$

TABLE I, continued.

Quantity	Symbol, equation	Value
pressureless matter density of the universe	$\Omega_{\mathrm{m}} = \Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}}$	$0.315^{+0.016}_{-0.017}$
CMB radiation density of the universe	Ω_{γ}	$2.473 \cdot 10^{-5} (T/2.7255\ \mathrm{K})^4 h^{-2} =$ $= 5.46(19) \cdot 10^{-5}$
curvature	$\Omega_{\mathrm{tot}} = \Omega_{\mathrm{m}} + \Omega_{\Lambda} + \cdots$	$0.96^{+0.4}_{-0.5}$ (95% CL); $1.000(7)$ (95% CL; CMB + BAO)
sum of neutrino masses	$\sum m_{\nu} c^2$	< 0.23 eV
neutrino density of the universe	Ω_{ν}	$< 0.0025\, h^{-2}$; < 0.0055
redshift of matter-radiation equality	z_{eq}	3360 ± 70
redshift of photon decoupling	z_{dec}	1090.2 ± 0.7
redshift at half reionization	z_{reion}	11.1 ± 1.1
age of the universe	t_0	13.81 ± 0.05 Gyr = $= (4.358 \pm 0.016) \cdot 10^{17}$ s

TABLE II. The measured, cosmological parameter values of Planck 2013 [11]. In column 1, the corresponding parameter symbols are given. The column 2 gives results for the Planck temperature data alone. Column 3, denoted as Planck+WP, combines these Planck data and the WMAP polarization data at low multipoles.

	Planck	**Planck+WP**
Parameter	**Best fit [68% limits]**	**Best fit [68% limits]**
H_0 [km/s Mpc]	67.11 [67.4 ± 1.4]	67.04 [67.3 ± 1.2]
Ω_Λ	0.6825 [0.686 ± 0.020]	0.6817 [$0.685^{+0.018}_{-0.016}$]
$100\,\Omega_b h^2$	2.2068 [2.207 ± 0.033]	2.2032 [2.205 ± 0.028]
$\Omega_{dm} h^2$	0.12029 [0.1196 ± 0.0031]	0.12038 [0.1199 ± 0.0027]
$\Omega_m h^2$	0.14300 [0.1423 ± 0.0029]	0.14305 [0.1426 ± 0.0025]
Age t_0 [Gyr]	13.819 [13.813 ± 0.058]	13.8242 [13.817 ± 0.048]
z_{eq}	3402 [3386 ± 69]	3403 [3391 ± 60]
z_{dec}	1090.43 [1090.37 ± 0.65]	1090.48 [1090.43 ± 0.54]
z_{reion}		11.37 [11.1 ± 1.1]

TABLE III. Important, known properties of particles and antiparticles.

Property	**Particle**	**Anti-particle**
energy	E	E
mass	M	M
spin	s	s
momentum	p	$-p$
velocity	v	$-v$
elementary electric charge	q	$-q$

TABLE IV. Most important quantities of the massless and massive universe and anti-universe, using by us the CPT symmetry[a)] for particle and antiparticles.

Quantity	**Universe**	**Anti-universe**
energy	E	E
temperature	T	T
Boltzmann constant	k	k
gravitational constant	G_N	G_N
age	t_0	t_0
distance, scale factor	R	$-R$
momentum	p	$-p$
velocity including c	v	$-v$

TABLE IV, continued.

Quantity	Universe	Anti-universe
reduced Planck constant	$\hbar$	$-\hbar$
time	t	$-t$
Hubble expansion rate	H	$-H$
scale factor of H	h	$-h$
time difference	Δt	$-\Delta t$
ionized charges	q	$-q$
particle horizon	d	$-d$

[a)] For this Table IV, we have also used the CPT symmetry [7], which exchanges simultaneously particles with antiparticles, connected with a change of sign of the three-dimensional coordinate vectors in combination with a change of the direction of the time. Besides, according to the CPT symmetry, the mass and the lifetime of particles and antiparticles are the same, whereas their magnetic dipole moments and momentums have opposite sign.

Table V. The estimated, particle-defined, (present) cosmological parameter values for universe and anti-universe according to Refs. [1-3] (compare with Tables I and II). [a)] Universe: "+"sign, anti-universe: "$-$"sign.

Symbol, equation	Value for universe and anti-universe
$\lvert\pm h\rvert$ [a)]	$0.6736^{+0.0105}_{-0.0096}$
$\lvert\pm H_0\rvert$ [a)]	$100\,h\ \mathrm{km\ s^{-1}Mpc^{-1}} = 67.36^{+1.05}_{-0.96}\ \mathrm{km\ s^{-1}Mpc^{-1}} =$ $= h\times(9.777752\,\mathrm{Gyr})^{-1} = (2.183^{+0.034}_{-0.031})\cdot 10^{-18}\ \mathrm{s^{-1}}$

Table V (continued)

Symbol, equation	Value for universe and anti-universe
$\rho_{0C} = 3H_0^2/8\pi G_N$	$(4.78^{+0.15}_{-0.14})\cdot 10^3 (\mathrm{eV}/c^2)\ \mathrm{cm}^{-3}$
t_0	$13.82^{+0.06}_{-0.07}$ Gyr
Ω_{tot}	1
$\Omega_{\mathrm{b}} = \rho_{\mathrm{b}}/\rho_{0C}$	$0.02211^{+0.00089}_{-0.00091}\, h^{-2} = 0.0487 \pm 0.0020$
$\Omega_{\mathrm{dm}} = \rho_{\mathrm{dm}}/\rho_{0C}$	$0.1202^{+0.0059}_{-0.0051}\, h^{-2} = 0.265^{+0.013}_{-0.012}$
$\Omega_{\Lambda} = \rho_{\Lambda}/\rho_{0C}$	$0.311^{+0.014}_{-0.013}\, h^{-2} = 0.686^{+0.020}_{-0.021}$
$\Omega_{\mathrm{m}} = \rho_{\mathrm{m}}/\rho_{0C}$	$0.1423^{+0.0059}_{-0.0052}\, h^{-2} = 0.314^{+0.013}_{-0.012}$
$\Omega_{\gamma} = \rho_{\gamma}/\rho_{0C}$	$(2.4728 \pm 0.0025)\cdot 10^{-5} h^{-2} = (5.45^{+0.15}_{-0.17})\cdot 10^{-5}$
Ω_{ν}	$(6.35^{+0.16}_{-0.14})\cdot 10^{-4} h^{-2}$; $\Rightarrow$ $(1.402^{+0.040}_{-0.044})\cdot 10^{-3}$
$\sum m_{\nu} c^2$	$(5.97^{+0.14}_{-0.13})\cdot 10^{-2}$ eV
$z_{\mathrm{eq}} \simeq \dfrac{2\Omega_{\mathrm{m}}}{N(T)\Omega_{\gamma}} - 1$	3423^{+107}_{-94} (see Eq. (7.5))
z_{dec}	1090.7 (see Eq. (4.19))
$z_{\mathrm{reion}}(\nu_{\tau})$	$11.60^{+0.31}_{-0.26}$ (see Eq. (7.45))

In contrast to Ref. [1], for Tables IV and V, we have corrected the corresponding transformations between the universe and the anti-universe at the scale factor as well as the age.

More generally, at the Friedmann-Lemaitre Equations (2.3) to (2.5), for the Standard model ($\Lambda = 0$), it is usual to introduce the mass density

$$\rho = \rho_\Lambda + \rho_\mathrm{m} + \rho_\mathrm{r} = \left[\Omega_\Lambda + \Omega_\mathrm{m}(R_0/R)^3 + \Omega_\mathrm{r}(R_0/R)^4\right]\rho_{0\mathrm{C}} =$$

$$= \left[\Omega_\Lambda + \Omega_\mathrm{m}(1+z)^3 + \Omega_\mathrm{r}(1+z)^4\right]\frac{3H_0^2}{8\pi G_N}, \tag{2.6}$$

where according to Table I $\rho_{0\mathrm{C}}$ is the present, critical density

$$\rho_{0\mathrm{C}} = \frac{3H_0^2}{8\pi G_N}. \tag{2.7}$$

The radiation density parameter Ω_r is defined by

$$\Omega_\mathrm{r} = \frac{1}{2}N(T)\,\Omega_\gamma, \tag{2.8}$$

where the statistical particle weights $N(T)$ are summarized in Table VI. In Eq. (2.8), for temperatures $T << 1\,\mathrm{MeV}$, $N(T)$ has the value

$$N(T) = 3.362644, \tag{2.9}$$

since the only relativistic species are photons at temperature T as well as massless or nearly massless neutrinos and antineutrinos of three different types at the temperature $T_\nu = (4/11)^{1/3}T$, i.e. $N(T)$ of Eq. (2.8) has the value 3.362644 at the present CMB temperature $T_0 = 2.7255\,\mathrm{K}$ (see Table I) and increases at higher temperatures (see Table VI). Thus, we expect that the present, massive universe contains a relic background of light neutrinos with the temperature

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_0 = 1.9454\ \mathrm{K}. \tag{2.10}$$

Then, by Eqs. (2.6) and (2.7), the relativistic formalism (2.5) yields

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_\mathrm{m}\left(\frac{R_0}{R}\right)^3 + \Omega_\mathrm{r}\left(\frac{R_0}{R}\right)^4\right] \tag{2.11}$$

with the general solution

$$t = t(z) = \frac{1}{H_0}\int_0^{1/(1+z)} \frac{dx}{x\,(\Omega_\Lambda + \Omega_\mathrm{m}x^{-3} + \Omega_\mathrm{r}x^{-4})^{1/2}}, \tag{2.12}$$

where $x \equiv (R/R_0) = (1/[1+z])$. In Eq. (2.12), for a flat universe, at $z \to 0$, because of the very small value Ω_r compared with Ω_Λ and Ω_m, the term $\Omega_r x^{-4}$ can be neglected, since at present most of the integral (2.12) comes from an epoch when the expansion is dominated by the matter and the dark energy. Thus, by Eq. (2.12), for $z = 0$, the age of the present universe (see Table I) is defined by

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{dx}{x\,(\Omega_\Lambda + \Omega_m x^{-3})^{1/2}} . \tag{2.13}$$

It is usual to take apart Eq. (2.6) in the dominant density terms in dependence on the redshift evolution.

Then, for an universe, which expands adiabatically, the entropy per comoving volume is constant, so that we can describe the mass density of the radiation-dominated universe for $z \geq 10^5$ ($T \geq 3 \cdot 10^5$ K) by

$$\rho = \rho_r = \{\pi^2/30\,(\hbar^3 c^5)\}\,(kT)^4 N(T) , \tag{2.14}$$

where $\hbar$ is the Planck constant and k is the Boltzmann constant.

For Eq. (2.14), the values of $N(T)$ are given in Table VI. At higher temperatures, these values depend on the particle physics models [9]. Because of Table III, we can assume that they are also valid for anti-particles, i.e. together with Table V we can also describe the radiation-dominated anti-universe.

Then, taking the known connections

$$\rho_0(\gamma) = \{\pi^2/15\,(\hbar^3 c^5)\}\,(kT_0)^4 \tag{2.15}$$

and

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0C}} = 5.46 \cdot 10^{-5} , \tag{2.16}$$

for Eq. (2.14), we obtain

$$\rho = \rho_r = \frac{1}{2} N(T)\, \Omega_\gamma \rho_{0C} \left(\frac{T}{T_0}\right)^4 = \Omega_r \rho_{0C} \left(\frac{T}{T_0}\right)^4 . \tag{2.17}$$

TABLE VI. The statistical particle weights $N(T)$, taken from Ref. [9]. [a)] T_c corresponds to the confinement-deconfinement transition between quarks and hadrons. [b)] For $T > m_{\mathrm{X,Y}}$, the minimal SU(5) model (SUSY GUT) yields $N(T) = 160.75$.

Temperature	**New particles**	$4N(T)$ [b)]
$T < m_{\mathrm{e}}$	photons (γ)+neutrinos (ν)	29
$m_{\mathrm{e}} < T < m_{\mu}$	$\mathrm{e}^{\pm}$	43
$m_{\mu} < T < m_{\pi}$	$\mu^{\pm}$	57
$m_{\pi} < T < T_c$ [a)]	$\pi^{\pm}$, π^0	69
$T_c < T < m_{\mathrm{strange}}$	$\pi^{\pm}$, $\pi^0 + u, \bar{u}, d, \bar{d}$ + gluons	205
$m_s < T < m_{\mathrm{charm}}$	$s, \bar{s}$	247
$m_c < T < m_{\tau}$	$c, \bar{c}$	289
$m_{\tau} < T < m_{\mathrm{bottom}}$	$\tau^{\pm}$	303
$m_b < T < m_{\mathrm{W,Z}}$	$b, \bar{b}$	345
$m_{\mathrm{W,Z}} < T < m_{\mathrm{Higgs}}$	$\mathrm{W}^{\pm}$, Z^0	381
$m_H < T < m_{\mathrm{top}}$	H^0	385
$m_t < T$	$t, \bar{t}$	427

Comparing the corresponding density terms in Eqs. (2.6) and (2.17), via Eq. (2.2), for relativistic particles, we get

$$1+z=\frac{R_0}{R}=\frac{T}{T_0}. \tag{2.18}$$

Using the results (2.14) and (2.18), the Friedmann equation (2.5) yields uniquely the time for the radiation-dominated universe ($z \geq 10^5$) as a function of the thermal energy kT, the temperature T or the redshift z by the expressions

$$t=t(z)=\frac{1}{2}\left(\frac{90\hbar^3c^5}{8\pi^3G_N}\right)^{1/2}\frac{1}{\sqrt{N(T)}\,(kT)^2}=$$

$$=\frac{(2.42035\pm0.00015)}{\sqrt{N(T)}}\left(\frac{\text{MeV}}{kT}\right)^2\text{s}=$$

$$=\frac{(3.25936\pm0.00021)}{\sqrt{N(T)}}\left(\frac{10^{10}\,\text{K}}{T}\right)^2\text{s}=$$

$$=\frac{(0.438773\pm0.000028)}{\sqrt{N(T)}}\left(\frac{10^{10}}{1+z}\right)^2\text{s}. \tag{2.19}$$

According to Eqs. (2.2) and (2.6), for the matter-dominated universe, at $z_{\text{eq}}=3360>>z>>1$ (see Tables I and II), we observe as dominant density

$$\rho=\rho_{\text{m}}=\left[\Omega_{\text{m}}(R_0/R)^3\right]\frac{3H_0^2}{8\pi G_N}=\left[\Omega_{\text{m}}(1+z)^3\right]\frac{3H_0^2}{8\pi G_N}, \tag{2.20}$$

whereas for $z_{\text{eq}}>>z\to1$ we cannot more neglect the dark mass density $\Omega_\Lambda\rho_{0\text{C}}=3\Omega_\Lambda H_0^2/8\pi G_N$, so that here we have

$$\rho=\rho_\Lambda+\rho_{\text{m}}=\left[\Omega_\Lambda+\Omega_{\text{m}}(R_0/R)^3\right]\frac{3H_0^2}{8\pi G_N}=$$

$$=\left[\Omega_\Lambda+\Omega_{\text{m}}(1+z)^3\right]\frac{3H_0^2}{8\pi G_N}. \tag{2.21}$$

In Sec. 7.1, these cases (2.20) and (2.21) are treated.

If in Eq. (2.6) the dark mass density

$$\rho=\rho_\Lambda=\Omega_\Lambda\rho_{0\text{C}} \tag{2.22}$$

is dominant, Eq. (2.5) yields the present, exponential or accelerated expansion

$$R = R_0\, \mathrm{e}^{\Omega_\Lambda^{1/2} H_0\,(t-t_0)} = R_0\, \mathrm{e}^{H(t-t_0)} \quad \text{at} \quad H = \Omega_\Lambda^{1/2} H_0\,, \tag{2.23}$$

where t_0 is the age of the present-day universe (see Table I).

If there is an universe for $\Lambda \neq 0$ with a dominant source of constant vacuum (vac) mass density ρ_{vac}, we would observe this mass density as cosmological "constant" $\Lambda = 8\pi G_{\mathrm{N}}\, \rho_{\mathrm{vac}} / c^2$. In this case (denoted in this work as old inflation model (see Sec. 3.2.5), Eq. (2.5) yields an extremely rapid, exponential expansion of the early universe according to

$$R = R_1\, \mathrm{e}^{\sqrt{8\pi G_N \rho_{\mathrm{vac}}/3}\,\Delta t} = R_1\, \mathrm{e}^{\sqrt{c^2\Lambda/3}\,\Delta t} = R_1\, \mathrm{e}^{H\Delta t}\,, \tag{2.24}$$

where Δt describes thus the period of the inflationary phase, whereas H denotes the short-time Hubble constant.

For this early universe, the supersymmetric grand unification theories (SUSY GUTs) predict that X, Y gauge bosons and magnetic monopoles play an important role as a result of the unification of the strong and electroweak interaction at energies $E_{\mathrm{G}} \approx 3 \cdot 10^{16}\,\mathrm{GeV}$ [6]. In these SUSY GUTs, the baryon number is violated, i.e. we must expect a proton decay [6]. The rest energy of the X and Y gauge bosons is equivalent to $E_{\mathrm{G}} \approx 3 \cdot 10^{16}\,\mathrm{GeV}$, whereas the rest energy of the magnetic monopoles (M) is given by $E_0(\mathrm{M}) = E_{\mathrm{G}} / \alpha_{\mathrm{GUT}}$ [6, 12], where the SUSY coupling constant is $\alpha_{\mathrm{GUT}} \approx 0.04$ [13-17].

Consequently, the standard cosmology ($\Lambda = 0$) provides a very successful framework for the explanation of the cosmic observations [1-5]. However, there are a number of problems, which cannot simply be solved within the standard cosmology. These problems are the horizon problem [1-5, 7], the monopole problem [1-5, 7], the flatness problem [1-5, 7] and the initial condition problem [1-5]. A way out of these problems is the inflation [1-5, 7], which in the early universe enlarges the scale factor by a factor of about 10^{30} [1-4, 7, 8, 10]. However, this inflationary cosmology and its transition to the standard model are quantitatively still an open problem [1-4, 8].

Using the data of Tables III to V, the considerations, treated in this Sec. 2 for the massive universe, are also valid for the massive anti-universe. To this problem, we will also return in the next chapters.

3 The light neutrinos as well as SUSY GUT particles and the new inflation model including its vacuum energy density

3.1 The light neutrino data

3.1.1 The rest energy of the light neutrinos

There are three light (left-handed) neutrinos (tauon neutrino ν_τ, muon neutrino ν_μ and electron neutrino ν_e).

According to Beringer et al. [9], the best values of the 3-neutrino oscillation parameters, derived from global neutrino oscillation data, yield for the square mass differences of the neutrino rest energies [1-4]:

$$\Delta m_{\tau\mu}^2 c^4 = E_0^2(\nu_\tau) - E_0^2(\nu_\mu) = (2.35^{+0.12}_{-0.09}) \cdot 10^{-3}\ \mathrm{eV}^2, \tag{3.1}$$

$$\Delta m_{\mu e}^2 c^4 = E_0^2(\nu_\mu) - E_0^2(\nu_e) = (7.58^{+0.22}_{-0.26}) \cdot 10^{-5}\ \mathrm{eV}^2. \tag{3.2}$$

From Eqs. (3.1) and (3.2), we can form the equations

$$E_0(\nu_\tau) = \left(\Delta m_{\tau\mu}^2 c^4 + E_0^2(\nu_\mu)\right)^{1/2} = \\ = \left(\Delta m_{\tau\mu}^2 c^4 + \Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2} \tag{3.3}$$

and

$$E_0(\nu_\mu) = \left(\Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2}. \tag{3.4}$$

By Beringer et al., for the normal hierarchy $E_0(\nu_\tau) > E_0(\nu_\mu) > E_0(\nu_e)$, from Eqs. (3.1) to (3.4), the neutrino rest energies $E_0(\nu_\tau)$ and $E_0(\nu_\mu)$ were estimated to

$$E_0(\nu_\tau) \approx \left|\Delta m_{\tau\mu}^2 c^4\right|^{1/2} \approx 4.8 \cdot 10^{-2}\,\mathrm{eV} \tag{3.5}$$

and

$$E_0(\nu_\mu) \approx \left|\Delta m_{\mu e}^2 c^4\right|^{1/2} \approx 8.6 \cdot 10^{-3}\,\mathrm{eV}. \tag{3.6}$$

Then, for normal hierarchy, because of transformations of various types of neutrinos into one another (neutrino oscillations), their rest energies must be connected [1-4], i.e. we can assume as new approach

$$E_0(\nu_e)/E_0(\nu_\mu) = E_0(\nu_\mu)/E_0(\nu_\tau) \quad \Rightarrow \quad E_0(\nu_e) = \frac{E_0^2(\nu_\mu)}{E_0(\nu_\tau)}, \tag{3.7}$$

so that by Eqs. (3.3) and (3.4) via Eq. (3.7) we obtain

$$E_0(\nu_e) = \frac{\Delta m_{\mu e}^2 c^4}{\left[\Delta m_{\tau\mu}^2 c^4 - \Delta m_{\mu e}^2 c^4\right]^{1/2}} = (1.589^{+0.078}_{-0.098}) \cdot 10^{-3}\,\mathrm{eV}. \tag{3.8}$$

Consequently, by Eq. (3.8), the expressions (3.3) and (3.4) yield

$$E_0(\nu_\tau) = \left(\Delta m_{\tau\mu}^2 c^4 + \Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2} \cong (4.93^{+0.12}_{-0.10}) \cdot 10^{-2}\,\mathrm{eV} \tag{3.9}$$

and

$$E_0(\nu_\mu) = \left(\Delta m_{\mu e}^2 c^4 + E_0^2(\nu_e)\right)^{1/2} \cong (8.85^{+0.14}_{-0.16}) \cdot 10^{-3}\,\mathrm{eV} \tag{3.10}$$

in accordance with Eqs. (3.5) and (3.6). Then, the sum of the neutrino rest energies (see Eqs. (3.8) to (3.10)) gives the approximate solution

$$\sum_i E_0(\nu_i) = E_0(\nu_\tau) + E_0(\nu_\mu) + E_0(\nu_e) \cong (5.97^{+0.14}_{-0.13}) \cdot 10^{-2}\,\mathrm{eV}, \tag{3.11}$$

where the subscript $i = \mathrm{e}, \mu, \tau$ characterizes the e, μ and τ neutrino.

The sum (3.11) does not contradict the observed limit $\sum_i E_0(\nu_i) < 0.23\,\mathrm{eV}$ [6, 11].

Using the data of Table III, the results, derived in this Sec. 3.1.1 for the neutrinos of the universe, are also valid for the antineutrinos of the anti-universe.

3.1.2 The light neutrino density parameters of the universe

In this chapter, we derive still the neutrino densities of the universe. Prior to the derivation of these light neutrino density parameters, we must explain the connection between the radiation and the critical density of the present universe. These considerations are based on Refs. [1-4, 7, 9, 10]. The cosmic microwave (radiation) background (CMB) is observed as a black-body spectrum with the mean temperature

$$T = T_0 = 2.7255 \pm 0.0006\ \mathrm{K}\,. \tag{3.12}$$

Consequently, this observation is consistent with a Planck distribution at the temperature (3.12). The Planck distribution of photons is quantum statistically characterized by the energy density

$$\rho(\gamma)c^2 = \frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3} =$$

$$= (4.722158 \pm 0.000017)\cdot 10^{-3}\left(\frac{T}{\mathrm{K}}\right)^4 \mathrm{eV/cm^3} \tag{3.13}$$

and the number density

$$n(\gamma) = \frac{2.404113806}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 =$$

$$= (20.286843 \pm 0.000057)\left(\frac{T}{\mathrm{K}}\right)^3 \mathrm{cm}^{-3}\,, \tag{3.14}$$

where the value 2.404113806 corresponds to the integral $\int_0^\infty [x^2/(e^x - 1)]\mathrm{dx}$.

According to Eq. (2.16), the radiation density of the universe is defined by

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0\mathrm{C}}}\,, \tag{3.15}$$

where the present photon density $\rho_0(\gamma)$ and the critical present density $\rho_{0\mathrm{C}}$ are again given by

$$\rho_0(\gamma) = \frac{\pi^2}{15}\frac{(kT_0)^4}{\hbar^3 c^5} \qquad \text{(see Eqs. (2.15) or (3.13))}\,, \tag{3.16}$$

$$\rho_{0C} = \frac{3H_0^2}{8\pi G_N} \qquad \text{(see Eq. (2.7))}. \tag{3.17}$$

Thus, Eq. (3.16) gives

$$\rho_0(\gamma)\, c^2 = \frac{\pi^2}{15}\frac{(kT_0)^4}{(\hbar c)^3} = (0.26057 \pm 0.00023)\ \mathrm{eV\,cm^{-3}}. \tag{3.18}$$

For Eq. (3.17), according to Table I, we get

$$\rho_{0C}\, c^2 = \frac{3c^2 H_0^2}{8\pi G_N} = (1.05375 \pm 0.00013)\cdot 10^4 h^2\ \mathrm{eV\,cm^{-3}}. \tag{3.19}$$

Then, via Eqs. (3.18) and (3.19), Eq. (3.15) yields the present radiation density of the universe to

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0C}} = (2.4728 \pm 0.0025)\cdot 10^{-5} h^{-2}. \tag{3.20}$$

By Eqs. (3.13) and (3.14), the kinetic (K) energy of the photons is given by

$$E_K(\gamma) = \frac{\rho(\gamma)\, c^2}{n(\gamma)} = 2.701178\, kT. \tag{3.21}$$

Now, analogously to the present baryon number density [1-4, 6, 7], we can introduce the present neutrino number densities

$$n_0(\nu_i) = \Omega_\nu(\nu_i)\frac{\rho_{0C} c^2}{E_0(\nu_i)}, \tag{3.22}$$

so that the neutrino densities of the universe are determined by

$$\Omega_\nu(\nu_i) = E_0(\nu_i)\frac{n_0(\nu_i)}{\rho_{0C} c^2}. \tag{3.23}$$

For the neutrino temperature $T_0(\nu_i) = (4/11)^{1/3} T_0 = (1.94537 \pm 0.00043)\ \mathrm{K}$ [7, 9], we find the present neutrino number densities [1-4] (for every subscript $i = \mathrm{e}, \mu, \tau$) to

$$n_0(\nu_i) = \frac{3}{4}\frac{2.404113806}{\pi^2}\frac{(k[(4/11)^{1/3} T_0])^3}{(\hbar c)^3} =$$

$$= (112.016 \pm 0.074)/\mathrm{cm}^3. \tag{3.24}$$

Analogous to Refs. [1-4, 6], the expressions (3.15) as well as (3.23) and (3.24) yield

$$\Omega_\nu(\nu_i) = E_0(\nu_i)\frac{n_0(\nu_i)}{\rho_0(\gamma)c^2}\Omega_\gamma = \frac{45}{11}\frac{2.4041138}{\pi^4}\frac{E_0(\nu_i)}{kT_0}\Omega_\gamma =$$
$$= (429.889 \pm 0.095)\,\mathrm{eV}^{-1}E_0(\nu_i)\,\Omega_\gamma =$$
$$= (0.010630 \pm 0.000011)\,\mathrm{eV}^{-1}E_0(\nu_i)\,h^{-2}. \qquad (3.25)$$

Using the rest energies (3.8) to (3.10) together with the radiation density $\Omega_\gamma = (5.46 \pm 0.19)\cdot 10^{-5}$ (see Table I), by Eq. (3.25), we get (see also Refs. [1-4]):

$$\Omega_\nu(\nu_e) \cong (0.683^{+0.034}_{-0.042})\Omega_\gamma =$$
$$= (1.69^{+0.09}_{-0.11})\cdot 10^{-5}h^{-2} = (3.73^{+0.32}_{-0.36})\cdot 10^{-5}, \qquad (3.26)$$

$$\Omega_\nu(\nu_\mu) \cong (3.805^{+0.061}_{-0.070})\Omega_\gamma =$$
$$= (9.41^{+0.16}_{-0.18})\cdot 10^{-5}h^{-2} = (2.08 \pm 0.11)\cdot 10^{-4}, \qquad (3.27)$$

$$\Omega_\nu(\nu_\tau) \cong (21.19^{+0.52}_{-0.44})\Omega_\gamma =$$
$$= (5.24^{+0.13}_{-0.11})\cdot 10^{-4}h^{-2} = (1.157^{+0.069}_{-0.064})\cdot 10^{-3}. \qquad (3.28)$$

The sum of Eqs. (3.26) to (3.28) gives

$$\Omega_\nu = \sum_i \Omega_\nu(\nu_i) \cong (25.68^{+0.62}_{-0.55})\Omega_\gamma =$$
$$= (6.35^{+0.16}_{-0.14})\cdot 10^{-4}h^{-2} = (1.402^{+0.083}_{-0.079})\cdot 10^{-3}. \qquad (3.29)$$

The sum (3.29) does not contradict the observed limiting values $\Omega_\nu < 0.0055$ and $\Omega_\nu < 0.0025h^{-2}$ (see Table I). Then, the deviations amount to only a factor of 4.

Using the data of Tables III to V, the results, derived in this Sec. 3.1.2 for the universe, are also valid for the anti-universe.

3.2 The SUSY GUT particles as well as the new inflation model and its vacuum energy density

3.2.1 The necessary properties of the X and Y gauge bosons

For the Standard $\mathrm{SU}(2)\times\mathrm{U}(1)$ electroweak theory, Fujikawa and Shrock [18] (see also Ref. [9]) have derived the magnetic moment of massive neutrinos to

$$\mu_\nu(\nu_i)=\frac{3}{8\pi^2\sqrt{2}}\,e[G_F/(\hbar c)^3]\hbar c^2\,E_0(\nu_i)=$$
$$=3.203\cdot10^{-19}(E_0(\nu_i)/\mathrm{eV})\,\mu_B\,, \tag{3.30}$$

where e and $[G_F/(\hbar c)^3]=1.1663787\cdot10^{-23}\mathrm{eV}^{-2}$ describes the elementary charge and the Fermi coupling constant, respectively [9]. The Bohr magneton μ_B [9] is defined by

$$\mu_B=\frac{e\hbar c^2}{2E_0(\mathrm{e})}=(5.7883818066(38)\cdot10^{-11}\,\mathrm{MeV/T}\,, \tag{3.31}$$

where $E_0(\mathrm{e})$ is the rest energy of the electron (e) [9]. The magnetic moment of the electron is given by $\mu_\mathrm{e}=\mathrm{C_e}\mu_B$, where $\mathrm{C_e}\cong1.00116$ [9]. Similar to the nucleon magneton $\mu_N=(E_0(\mathrm{e})/E_0(p))\mu_B$ [9] with the proton (p) rest energy $E_0(\mathrm{p})$ [9], the magneton of the X gauge boson (SUSY grand unification magneton [1-4]) is assumed to

$$\mu_\mathrm{X}=\frac{E_0(\mathrm{e})}{E_0(\mathrm{X})}\mu_B\,, \tag{3.32}$$

where $E_0(\mathrm{X})=E_0(\mathrm{Y})$ is the rest energy of the X, Y gauge bosons, so that analogous to the magnetic moment of the electron (see above) the magnetic moment of the neutrinos, using reasonably $1\le\mathrm{C}_\nu\le\mathrm{C_e}$, may be assumed to

$$\mu_\nu(\nu_i)\cong\mathrm{C}_\nu\frac{E_0(\nu_i)}{\sum_i E_0(\nu_i)}\mu_\mathrm{X}=\mathrm{C}_\nu\frac{E_0(\nu_i)}{\sum_i E_0(\nu_i)}\frac{E_0(\mathrm{e})}{E_0(\mathrm{X})}\mu_B\,. \tag{3.33}$$

Then, the derivation of Eq. (3.33) is clear because the expression $\sum_i\mu_\nu(\nu_i)/\mathrm{C}_\nu$ gives again the supersymmetric grand unification magneton

μ_X (see Eq. (3.32)). By Eqs. (3.30) and (3.33), if we assume as the maximum value $C_\nu = C_e$ in Eq. (3.33), we may estimate the rest energy of the X gauge boson to

$$E_0(\mathrm{X}) \cong \mathrm{C_e} \frac{E_0(\nu_i)}{\sum_i E_0(\nu_i)} \frac{E_0(\mathrm{e})}{\mu_\nu(\nu_i)} \mu_B =$$

$$= \mathrm{C_e} \frac{E_0(\mathrm{e})}{\sum_i E_0(\nu_i)} \frac{\mathrm{eV}}{3.203 \cdot 10^{-19}} = (2.675^{+0.058}_{-0.063}) \cdot 10^{16}\ \mathrm{GeV}. \qquad (3.34)$$

According to Sec. 2, for the early universe, the value (3.34) agrees excellently with the gauge coupling unification $E_G \approx 3 \times 10^{16}\,\mathrm{GeV}$ of SUSY GUTs [6], which is identical with $E_0(\mathrm{X}) = E_G$ and $E_0(\mathrm{Y}) = E_G$ [6, 12].

Because of the new interpretation $kT = E_0(\mathrm{X})$ of the thermal equilibrium (see Sec. I or the paragraph after Eq. (3.36)), by Eqs. (2.19) and (3.34) as well as $N(T) = N_{\mathrm{X,Y}}(T) = 160.75$ (see Table VI, which is also valid for the anti-universe because of Table III), we obtain the time $t_{\mathrm{X,Y}}$ and the temperature $T_{\mathrm{X,Y}}$ for these X and Y gauge bosons [1-4] of SUSY GUTs:

$$t = t_{\mathrm{X,Y}} = \frac{2.42035}{\sqrt{N_{\mathrm{X,Y}}(T)}} \left[\frac{\mathrm{MeV}}{E_0(\mathrm{X})}\right]^2 \mathrm{s} = (2.67^{+0.13}_{-0.12}) \cdot 10^{-40}\ \mathrm{s} \qquad (3.35)$$

and $$T = T_{\mathrm{X,Y}} = E_0(\mathrm{X})/k = (3.104^{+0.067}_{-0.073}) \cdot 10^{29}\ \mathrm{K} \propto t_{\mathrm{X,Y}}^{-1/2}\,. \qquad (3.36)$$

At the beginning of the massive universe ($R \geq R_{\mathrm{Pl}}$), all real particles (P) have the Planck energy as relativistic energy in the new thermal equilibrium [1-4] with the photons ($kT \geq E_0(\mathrm{P})$). In this particle mixture, only the events, which are determined by the reaction laws of nuclear and elementary physics, can take place, i.e. we have no complete mixture of particles and antiparticles in the universe, so that the small number of particles and antiparticles, produced by these reaction laws, must again disappear by annihilation. Therefore, in this work, we can also use the statistical weight $N(T)$, given in Sec. 2, since they were obtained by consideration of these reaction laws of nuclear and elementary particle physics.

Taking Eqs. (2.18) and (3.36), we find the redshift of the X and Y gauge bosons to

$$1+z_{\mathrm{X,Y}}=\frac{T_{\mathrm{X,Y}}}{T_0}=(1.139^{+0.025}_{-0.027})\cdot 10^{29}\,. \tag{3.37}$$

Using the data of Tables III to V, the results, derived in this Sec. 3.2.1 for the universe, are also valid for the anti-universe.

3.2.2 "Two scale factors" of absorption of light for the redshift condition

Using $H_0=(2.181\pm 0.039)\cdot 10^{-18}\,\mathrm{s}^{-1}$ (see Table I), for the present-day critical density $\rho_{0\mathrm{C}}$ (see Eq. (2.7)), we can introduce (see also Refs. [1, 2, 4]):

$$\rho_{0\mathrm{C}}=\frac{3H_0^2}{8\pi G_N}\left[\frac{R_{\mathrm{Pl}}^2}{R_{\mathrm{Pl}}^2}\right]=\frac{3H_0^2}{8\pi G_N}\left[\frac{\hbar\, G_N/c^3}{R_{\mathrm{Pl}}^2}\right]=$$
$$=\frac{3\hbar}{8\pi c(R_{\mathrm{Pl}}c/H_0)^2}=\frac{3\hbar}{8\pi c\,\widetilde{R}_0^4}\,, \tag{3.38}$$

where the Planck length $R_{\mathrm{Pl}}=1.616200\cdot 10^{-35}\,\mathrm{m}$ is defined by Eq. (7.1), so that the scale factor $\widetilde{R}_0$ is given by

$$\widetilde{R}_0=(R_{\mathrm{Pl}}c/H_0)^{1/2}=(4.713\pm 0.042)\cdot 10^{-5}\,\mathrm{m}\,. \tag{3.39}$$

Similar to Eq. (3.38), we can also introduce (see also Ref. [1, 2, 4]):

$$\rho_{0\mathrm{C}}=\frac{3H_0^2}{8\pi G_N}\left[\frac{c^2}{c^2}\right]=\frac{3}{8\pi G_N}\frac{c^2}{[c/H_0]^2}=\frac{3c^2}{8\pi G_N R_0^2} \tag{3.40}$$

with

$$R_0=c/H_0=(1.375\pm 0.025)\cdot 10^{26}\,\mathrm{m}\,. \tag{3.41}$$

Between Eqs. (3.39) and (3.41), we have trivially the connection $\widetilde{R}_0=(R_{\mathrm{Pl}}R_0)^{1/2}$.

Because of $\widetilde{R}_0 << R_0$, we assume that Eqs. (3.39) and (3.41) represent the "two" unknown scale factors of the absorption of light at the redshift (see Eqs. (2.2) or (2.18)) before and after the inflation [1-4], respectively.

Thus, to these values (3.39) and (3.41), we will return in Sec. 3.2.4, where they are used generally for the determination of the scale factors $R = R(t)$ by the redshift condition (2.2) or (2.18) via the new inflation model [1-4].

Using the data of Tables IV and V, all results, derived in this Sec. 3.2.2 for the universe, are also valid for the anti-universe.

3.2.3 The necessary properties of the magnetic monopoles

Using the two scale factors of the absorption of light at the redshift before (see Eq. (3.39)) and after (see Eq. (3.41)) the inflation (see also Sec. 3.2.4 as well as Refs. [1-4]), because of Eqs. (2.2) and (2.18), we can define

$$1 + z_{\mathrm{M}} = \frac{T_{\mathrm{M}}}{T_0} = \frac{\widetilde{R}_0}{R_{\mathrm{PI}}} = \frac{R_0}{\widetilde{R}_0} = (2.916 \pm 0.026) \cdot 10^{30} . \tag{3.42}$$

In Eq. (3.42), in contrast to Eq. (3.37), because of $E_0(\mathrm{M}) > E_0(\mathrm{X})$ (see the third paragraph from the end of Sec. 2), the subscript M is assumed to define the magnetic monopoles, so that via Eqs. (2.18) and (3.12) we can determine their temperature [1-4] because of the new thermal equilibrium (see Sec. I or the paragraph after Eq. (3.36) in Sec. 3.2.1) as follows:

$$T_{\mathrm{M}} = (1 + z_{\mathrm{M}}) T_0 = (7.948 \pm 0.073) \cdot 10^{30}\ \mathrm{K} . \tag{3.43}$$

Thus, the rest energy of the magnetic monopoles (see also Refs. [1-4]) is defined by the redshift condition $1 + z_{\mathrm{M}} = T_{\mathrm{M}} / T_0$ (see Eq. (3.42)) to

$$E_0(\mathrm{M}) = kT_{\mathrm{M}} = (1 + z_{\mathrm{M}}) kT_0 = (6.849 \pm 0.063) \cdot 10^{17}\ \mathrm{GeV} , \tag{3.44}$$

if we use the new interpretation of the thermal equilibrium (see above).

Then, by Eqs. (3.34) and (3.44), the coupling constant of the SUSY GUTs [1-4, 6] is found to

$$\alpha_{\mathrm{GUT}} = \frac{E_0(\mathrm{X})}{E_0(\mathrm{M})} = 0.0391^{+0.0012}_{-0.0013} \tag{3.45}$$

in excellent agreement with the value $\alpha_{\mathrm{GUT}} \approx 0.04$ [13-17], which was obtained by a completely other way [17]. Therefore, the results (3.34) and (3.45) confirm the assumption (3.43) and the estimation (3.11) as well as the solutions (3.39) and (3.41) as the unknown scale factor of the absorption of light at the redshift condition (see Eq. (2.2)) before and after the inflation, described by the new inflation model (see Sec. 3.2.4 as well as Refs. [1-4]).

In SUSY GUTs the baryon number is violated, so that they predict a proton decay. By Eqs. (3.34) and (3.45), the proton lifetime [1, 2, 4, 6] is estimated to

$$\tau_{\mathrm{p}} = \frac{\hbar}{2\alpha_{\mathrm{GUT}}^2 [E_0(\mathrm{p})]^5 / [E_0(\mathrm{X})]^4} = 4.803 \cdot 10^{36}\ \mathrm{yr} \tag{3.46}$$

in accordance with the experimental limiting value of the proton lifetime $\tau_{\mathrm{p}}(\mathrm{p} \to \pi^0 \mathrm{e}^+) \geq 1.0 \cdot 10^{34}\,\mathrm{yr}$ [9].

With that, the non-SUSY GUTs (e.g., non-SUSY SU(5)), which predict a proton lifetime $\tau_{\mathrm{p}} \approx 10^{31}\,\mathrm{yr}$ at $E_{\mathrm{G}} \approx 10^{15}\,\mathrm{GeV}$ [17], are ruled out by the non-observation of the proton decay in the allowed range [6].

Using Eqs. (2.19) and (3.44), for the magnetic monopoles, we can determine the time (see also Refs. [1-4]):

$$t_{\mathrm{M}} = \frac{2.42035}{\sqrt{N_{\mathrm{M}}(T)}} \left[\frac{\mathrm{MeV}}{kT_{\mathrm{M}}}\right]^2 \mathrm{s} = \frac{(5.160 \pm 0.095) \cdot 10^{-42}}{\sqrt{N_{\mathrm{M}}(T)}}\ \mathrm{s}\,. \tag{3.47}$$

Using the data of Tables III to V, the results, derived in this Sec. 3.2.3 for the universe, are also valid for the anti-universe.

3.2.4 The new inflation model for the massive universe and anti-universe

In this chapter, according to Refs. [1-4], we analyse the connections (3.42):

$$1 + z_{\mathrm{M}} = \frac{T_{\mathrm{M}}}{T_0} = \frac{\widetilde{R}_0}{R_{\mathrm{Pl}}} = \mathrm{e}^{H\Delta t} = (2.916 \pm 0.026) \cdot 10^{30} \tag{3.48}$$

and

$$1+z_{\mathrm{M}}=\frac{T_{\mathrm{M}}}{T_0}=\frac{R_0}{\widetilde{R}_0}=\mathrm{e}^{H\Delta t}=(2.916\pm 0.026)\cdot 10^{30}\,, \tag{3.49}$$

whereat the exponential term $\mathrm{e}^{H\Delta t}$ is considered in the next Sec. 3.2.5.

For this goal, we use the radiation-dominated universe ($z>10^5$). Then, by Eqs. (2.14) and (2.17) as well as (2.18), we can write

$$\rho_{\mathrm{r}}=\{\pi^2/30(\hbar^3c^5)\}(kT)^4N(T)=\frac{1}{2}N(T)\,\rho_0(\gamma)\left(\frac{T}{T_0}\right)^4=$$

$$=\frac{1}{2}N(T)\,\Omega_\gamma\,\rho_{0\mathrm{C}}\left(\frac{T}{T_0}\right)^4=$$

$$=\frac{1}{2}N(T)\,\Omega_\gamma\,\rho_{0\mathrm{C}}\left(\frac{R_0}{R}\right)^4=\frac{1}{2}N(T)\,\Omega_\gamma\,\rho_{0\mathrm{C}}\,(1+z)^4\,. \tag{3.50}$$

Assuming instead of $1+z=R_0/R$ (see Eq. (3.41)) the ratio $\widetilde{R}_0/\widetilde{R}=1+z$ (see Eq. (3.39)) and using $\rho_{0\mathrm{C}}$ (according to Eqs. (2.7) or (3.17)) in Eq. (3.50), Eq. (2.5) yields

$$\widetilde{R}=(2\,N(T)\,\Omega_\gamma)^{1/4}(R_{\mathrm{Pl}}\,c\,t)^{1/2}\propto t^{1/2}\,, \tag{3.51}$$

whereas for $1+z=R_0/R$ in Eq. (3.50) the Friedmann equation (see Eq. (2.5)) gives

$$R=(2\,N(T)\,\Omega_\gamma)^{1/4}(R_0\,c\,t)^{1/2}\propto t^{1/2}\,, \tag{3.52}$$

so that for the radiation-dominated universe ($z>10^5$) by Eqs. (3.51) and (3.52) we can derive

$$t=t(z)=\frac{1/H_0}{(2\,N(T)\,\Omega_\gamma)^{1/2}\,(1+z)^2}\quad (\mathrm{z}>10^5)\,. \tag{3.53}$$

At the matter-dominated universe, for Eqs. (2.20) and (2.21), the time $t=t(z)$ is hereafter determined by Eqs. (7.9) and (7.11), respectively.

Then, using $t=t_{\mathrm{M}}$ (see Eq. (3.47)) and $t=t_{\mathrm{X,Y}}$ (see Eq. (3.35)) in Eqs. (3.51) and (3.52), we can assume

$$\widetilde{R} = \widetilde{R}_{\mathrm{M}} = (2\,N_{\mathrm{M}}(T)\,\Omega_{\gamma})^{1/4}(R_{\mathrm{Pl}}\,c\,t_{\mathrm{M}})^{1/2} = R_{\mathrm{Pl}} = \frac{\widetilde{R}_0}{1+z_{\mathrm{M}}}\,, \tag{3.54}$$

$$R = R_{\mathrm{M}} = (2\,N_{\mathrm{M}}(T)\,\Omega_{\gamma})^{1/4}(R_0\,c\,t_{\mathrm{M}})^{1/2} = \widetilde{R}_0 = \frac{R_0}{1+z_{\mathrm{M}}}\,, \tag{3.55}$$

$$\widetilde{R} = \widetilde{R}_{\mathrm{X,Y}} = (2\,N_{\mathrm{X,Y}}(T)\,\Omega_{\gamma})^{1/4}(R_{\mathrm{Pl}}\,c\,t_{\mathrm{X,Y}})^{1/2} =$$

$$= \frac{\widetilde{R}_0}{1+z_{\mathrm{X,Y}}} = 4.138\cdot 10^{-34}\ \mathrm{m} \tag{3.56}$$

and

$$R = R_{\mathrm{X,Y}} = (2\,N_{\mathrm{X,Y}}(T)\,\Omega_{\gamma})^{1/4}(R_0\,c\,t_{\mathrm{X,Y}})^{1/2} =$$

$$= \frac{R_0}{1+z_{\mathrm{X,Y}}} = 1.207\cdot 10^{-3}\ \mathrm{m}\,. \tag{3.57}$$

Now, we can interpret Eqs. (3.48) and (3.49), using Eqs. (3.54) to (3.57). Then, Eq. (3.48) means that by the inflation the dimension R_{Pl} of the beginning of the early universe is enlarged by the enormous factor of $1+z_{\mathrm{M}} = 2.916\cdot 10^{30}$ to the scale factor $\widetilde{R}_0 = R_{\mathrm{Pl}} \times (1+z_{\mathrm{M}})$ of absorption of light at the redshift before the inflation, if we assume $\widetilde{R} = \widetilde{R}_0/(1+z)$ for every z value (see Eqs. (3.54) and (3.56)).

More generally, we have the connection $R = \widetilde{R}(1+z_{\mathrm{M}}) = R_0/(1+z)$ with the scale factor $R_0 = \widetilde{R}_0(1+z_{\mathrm{M}})$ of absorption of light for the redshift condition after the inflation (see Eqs. (3.49), (3.55) and (3.57)). Thus, indeed, because of Eqs. (3.54) to (3.57), the two scale factors $\widetilde{R}_0$ and R_0 are characteristical quantities (see Sec. 3.2.2).

Attention! The scale factor $\widetilde{R}_0$ has a double role because it determines the end of the early universe and the beginning of the late universe.

Therefore, for the redshift evolution $1+z$, we can generally introduce (see also Refs. [1-3]):

$$\widetilde{R} = \frac{R}{1+z_{\mathrm{M}}} = \frac{\widetilde{R}_0}{1+z} \tag{3.58}$$

and

$$R = (1 + z_{\mathrm{M}})\widetilde{R} = \frac{R_0}{1+z}, \tag{3.59}$$

where $\widetilde{R}$ and R characterize again the two scale factors for the emission of light at the redshift in the FRW geometry prior to and after the inflation, respectively.

Via the Planck time $t_{\mathrm{Pl}} = R_{\mathrm{Pl}}/c = (5.39106 \pm 0.00032) \cdot 10^{-44}\,\mathrm{s}$ (see Eq. (7.2)), the time $t_* = t_{\mathrm{Pl}}\,(1 + z_{\mathrm{M}}) = \widetilde{R}_0/c = (R_{\mathrm{Pl}}/cH_0)^{1/2} = 1.572 \cdot 10^{-13}\,\mathrm{s}$ (see Eqs. (3.39) and (3.41)) is a reduced Hubble time because of $1/H_0 = (1 + z_{\mathrm{M}})t_*$.

Then, using Eq. (3.58), we can also assume that because of $1 + z = T/T_0$ in the early space region $R_{\mathrm{Pl}} \le \widetilde{R} \le \widetilde{R}_0$ (see Eq. (3.48)) these redshift conditions for $T \le T_{\mathrm{M}}$ determine the expansion of this early, massive universe, which is converted into the late universe by the inflation according to Eq. (3.59).

In other words, the inflation enlarges the scale factors $\widetilde{R}$ of the early universe at $R_{\mathrm{Pl}} \le \widetilde{R} \le \widetilde{R}_0$ (see Eq. (3.58)) by the enormous factor $1 + z_{\mathrm{M}}$ (see Eq. (3.48)) to the scale factors R of the late universe for $\widetilde{R}_0 \le R \le R_0$ (see Eq. (3.59)), which has today the redshift $z = 0$ at a scale factor $R = R_0$.

By this above-introduced, new inflation model (see also Refs. [1-4]), we have also confirmed the solution of the pressing problems of the standard model as the horizon problem and the monopole problem, whereat because of the rest energy of the magnetic monopoles $E_0(\mathrm{M}) << E_{\mathrm{Pl}}$ (see Eqs. (3.44) and (7.3)) the initial condition problem can be treated by the classical cosmology.

According to Refs. [1-4], the flatness problem is generally solved by the derivation of the curvature $\Omega_{\mathrm{tot}} = 1$ or $\Omega_{\mathrm{tot}}(z) = 1$ with the parameter $\mathrm{k} = 0$ (see Eq. (2.5)).

Using the data of Tables III to V, the results, derived in this Sec. 3.2.4 for the universe, are also valid for the anti-universe.

3.2.5 Determination of the vacuum energy density by the new inflation model via quantities of the old inflation model for $R = R_{\mathrm{Pl}}$

Now, we consider the exponential term $\mathrm{e}^{H\Delta t}$ of the connections (3.48) and (3.49). The old inflation model (2.24) can be conserved into the new inflation model (see Sec. 3.2.4) if instead of the integration constant R_1 we assume formally the transformation $R_1 \to \widetilde{R}$, so that we obtain

$$R = \widetilde{R}\,\mathrm{e}^{H\Delta t} = \widetilde{R}\,(1 + z_{\mathrm{M}}) \quad \text{(see Eqs. (3.48) and (3.59))}\,, \tag{3.60}$$

where Δt describes again the inflationary phase (seen from $t = t_{\mathrm{Pl}}$), which is attributed to the era of the supersymmetric grand unification, whereas H is the short-time Hubble constant [1-4] at $R = R_{\mathrm{Pl}}$:

$$H = \left(\frac{8\pi G_N}{3}\rho_{\mathrm{vac}}\right)^{1/2} = \left(\frac{c^2\Lambda}{3}\right)^{1/2} \quad \text{(see Eq. (2.24))}\,. \tag{3.61}$$

In Eq. (3.60), the integration constant $\widetilde{R}$ must be taken into account for any, constant redshift $z \le z_{\mathrm{M}}$ (see Eqs. (3.58) and (3.59)), which is determined by the thermal energy $kT \le kT_{\mathrm{M}}$ of the relativistic particles (see Eq. (2.18) and Sec. 3.2.4).

In other words, in Eq. (2.24), the transformation $R_1 \to \widetilde{R}$ means that the inflation acts on the total, early universe, which is defined by the redshift condition (3.58) before the inflation, so that this inflation leads to the later universe, which is described by the redshift condition (3.59) after the inflation. Indeed, this assumption describes a completely new inflation model. This interpretation is possible because we have introduced the new solution (3.51) for the early universe ($R_{\mathrm{Pl}} \le \widetilde{R} \le \widetilde{R}_0$) in contrast to the known expression (3.52) for the range $R \ge \widetilde{R}_0$. Instead of the solution (3.51), for many years, it was assumed that in the standard model the initial conditions could arise from the quantum cosmology because at small dimensions the classical cosmology is no longer valid. A way out of this problem was the old inflation model (2.24) of the early 1980s because it operates at an energy scale that is much lower than the Planck energy, i.e. the cosmology can be treated

classically [5]. The old inflation model solves indeed qualitatively the most pressing problems within the standard model (see the paragraph before last in Sec. 2). However, this old inflation model cannot be explained in the framework of the known particle physics. The new inflation model, which describes qualitatively and quantitatively these most pressing problems within the standard model, does not shown this difficulty because of the expressions (3.60).

Then, because of Eq. (3.60), we have the condition

$$H\Delta t = H\,(t_{\mathrm{X,Y}} - t_{\mathrm{M}}) = \ln(1 + z_{\mathrm{M}}) = 70.148\,, \tag{3.62}$$

where $t_{\mathrm{X,Y}}$ and t_{M} are the times (3.35) and (3.47), respectively.

From the statistical particle quantities is only known $N_{\mathrm{X,Y}}(T) = 160.75$ (see Eq. (3.35) and Table VI), whereas $N_{\mathrm{M}}(T)$ of Eq. (3.47) must be determined via the dependence (3.54), i.e. we assume $t_{\mathrm{M}} = t_{\mathrm{Pl}}$, so that we obtain

$$N_{\mathrm{M}}(T) = \left(\frac{5.160 \cdot 10^{-42}\,\mathrm{s}}{t_{\mathrm{Pl}}}\right)^2 = 9161 \pm 337\,. \tag{3.63}$$

Then, for $t_{\mathrm{M}} = t_{\mathrm{Pl}}$, the corresponding scale factors (see Eqs. (3.54) and (3.55)) are defined by

$$\widetilde{R} = R_{\mathrm{Pl}} = (2\,N_{\mathrm{M}}(T)\,\Omega_{\gamma})^{1/4}(R_{\mathrm{Pl}}\,c\,t_{\mathrm{Pl}})^{1/2} \tag{3.64}$$

and

$$R = \widetilde{R}_0 = (2\,N_{\mathrm{M}}(T)\,\Omega_{\gamma})^{1/4}(R_0\,c\,t_{\mathrm{Pl}})^{1/2}\,, \tag{3.65}$$

so that we must assume (see also Eqs. (3.51) and (3.52)):

$$N_{\mathrm{M}}(T) = \frac{1}{2\Omega_{\gamma}}\,. \tag{3.66}$$

Using the expressions (3.63) and (3.66), in accordance with Table I, we find

$$\Omega_{\gamma} = (5.46 \pm 0.20) \cdot 10^{-5}\,. \tag{3.67}$$

By Eq. (3.62), because of $t_{\mathrm{M}} = t_{\mathrm{Pl}}$ (see above) and $t_{\mathrm{X,Y}} >> t_{\mathrm{Pl}}$, we find the short-time Hubble H constant and the cosmological constant Λ for $R = R_{\mathrm{Pl}}$ (see also Refs. [1-4]) to

$$H = \frac{\ln(1+z_{\mathrm{M}})}{t_{\mathrm{X,Y}} - t_{\mathrm{M}}} \cong \frac{1}{t_{\mathrm{X,Y}}} \ln(1+z_{\mathrm{M}}) = (2.63^{+0.14}_{-0.15}) \cdot 10^{41}\,\mathrm{s}^{-1} \tag{3.68}$$

and

$$\Lambda = \frac{3H^2}{c^2} \cong (2.31^{+0.25}_{-0.26}) \cdot 10^{66}\,\mathrm{m}^{-2}. \tag{3.69}$$

Thus, at $t = t_{\mathrm{X,Y}} = 2.67 \cdot 10^{-40}\,\mathrm{s}$ (see Eq. (3.35)), the supersymmetric grand unification is already separated into the strong and electroweak interaction.

Consequently, by Eq. (2.24), for the boundary $R = R_{\mathrm{Pl}}$ of the massive universe, the Planck vacuum energy density, which is connected with the cosmological constant (3.69), can be estimated to

$$\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2 = \frac{c^4}{8\pi G_N}\Lambda = (6.94^{+0.75}_{-0.78}) \cdot 10^{121}\,\mathrm{eV\,cm}^{-3}. \tag{3.70}$$

Therefore, for the very short time of $\Delta t = 2.67 \cdot 10^{-41}\,\mathrm{s}$, the inflation field acts as positive $\Lambda = 2.31 \cdot 10^{66}\,\mathrm{m}^{-2}$ at $R = R_{\mathrm{Pl}}$ and yields the kinetic energy for an extremely rapid, exponential expansion of the early universe (see above).

For example, at the smallest possible, early universe ($R = R_{\mathrm{Pl}}$ with $r = 1$ as the dimensionless, time-independent, comoving coordinate [1, 5, 7]), because of Euclidian geometry (see Eq. (2.5)), this kinetic energy E_{K} of the extremely rapid, exponential expansion can be estimated by

$$\begin{aligned} E_{\mathrm{K}} &\cong \rho_{\mathrm{vac}} c^2 \tfrac{4}{3}\pi (R_{\mathrm{Pl}}\,\mathrm{e}^{H\Delta t})^3 \cong \\ &\cong \rho_{\mathrm{vac}} c^2 \tfrac{4}{3}\pi \left(\left[\widetilde{R}_0/(1+z_{\mathrm{M}})\right]\mathrm{e}^{H\Delta t}\right)^3 = 3.05 \cdot 10^{115}\,\mathrm{eV}. \end{aligned} \tag{3.71}$$

These estimations are possible because the inflation (via the rest energy of the magnetic monopoles) operates at an energy that is much lower than the Planck energy, so that we can treat classically the gravitation. Consequently, the expression $R_{\mathrm{Pl}} = \widetilde{R}_0/(1+z_{\mathrm{M}})$ (see Eqs. (3.48) and (3.54)) and the assumption $t_{\mathrm{M}} = t_{\mathrm{Pl}}$ (see Eqs. (3.64) and (3.66)) mean that via the reduced Compton wavelength $R_{\mathrm{Pl}} = \hbar c/E_{\mathrm{Pl}}$ because of $R_{\mathrm{Pl}} = ct_{\mathrm{Pl}}$ the relativistic energy of the magnetic monopoles is given by

$$E = \hbar/t_{\mathrm{Pl}} = \hbar c/R_{\mathrm{Pl}} = E_{\mathrm{Pl}}\,. \tag{3.72}$$

Then, if the cooling $T_{\mathrm{Pl}} \to T_{\mathrm{M}}$ in the universe has taken place, for the relativistic energy of the X and Y gauge bosons [1-3], because of the conditions $E_0(\mathrm{X}) < E_0(\mathrm{M})$ and $E_0(\mathrm{Y}) < E_0(\mathrm{M})$ [see Eqs. (3.34) and (3.44)], the sum of their relativistic maximum energies is limited by

$$E(\mathrm{X}) + E(\mathrm{Y}) = E_0(\mathrm{M})\,, \tag{3.73}$$

so that the condition $E(\mathrm{X}) = E(\mathrm{Y})$ determines their relativistic maximum energies to

$$E(\mathrm{X}) = \tfrac{1}{2} E_0(\mathrm{M}) \tag{3.74}$$

and

$$E(\mathrm{Y}) = \tfrac{1}{2} E_0(\mathrm{M})\,. \tag{3.75}$$

Using the data of Tables III and IV, the vacuum energy density of the anti-universe for $R = -R_{\mathrm{Pl}}$ has the same value as that of the universe (see Eqs. (3.68) to (3.70)), i.e. we have again

$$\rho_{\mathrm{vac}}(-R_{\mathrm{Pl}})c^2 = \frac{c^4}{8\pi G_N}\Lambda = 6.94 \cdot 10^{121}\ \mathrm{eV\,cm^{-3}}\,. \tag{3.76}$$

Finally, we show still that the cosmological "constant" is a variable. Then, from Eq. (2.6), for very small redshift conditions, a dominant, constant dark mass density term (see, e.g., Eqs. (2.7) and (3.40)) can be introduced as usually by

$$\rho = \rho_\Lambda = \Omega_\Lambda \rho_{0C} = \Omega_\Lambda \frac{3H_0^2}{8\pi G_N} = \Omega_\Lambda \frac{3c^2}{8\pi G_N R_0^2}\,. \tag{3.77}$$

According to quantum field theory [8], this dark mass density is connected with the vacuum mass density

$$\rho_{\mathrm{vac}} = \frac{c^2}{8\pi G_N}\Lambda = \rho_\Lambda = \Omega_\Lambda \frac{3c^2}{8\pi G_N R_0^2}\,, \tag{3.78}$$

which determines the present, accelerated expansion (see Eq. (2.24)), so that for $\Omega_\Lambda = 0.685^{+0.017}_{-0.016}$ (see Table I) the present cosmological "constant" Λ at $R = R_0 = (1.375 \pm 0.025) \cdot 10^{26}\,\mathrm{m}$ (see Eq. (3.41)) is found to

$$\Lambda = \frac{3\Omega_\Lambda}{R_0^2} = (1.087^{+0.067}_{-0.065}) \cdot 10^{-52}\,\mathrm{m}^{-2} \tag{3.79}$$

at a value of the vacuum energy density

$$\rho_{\mathrm{vac}} c^2 = \Omega_\Lambda \frac{3 H_0^2 c^2}{8\pi G_N} = (3.27 \pm 0.20) \cdot 10^3\,\mathrm{eV\,cm}^{-3}. \tag{3.80}$$

Considering Eqs. (3.77) to (3.80), we see that the vacuum energy density is again represented by the cosmological "constant", which has the value (3.79). Besides, it is connected with the dark energy [1-4], which is responsible for the present, accelerated expansion (see Eq. (2.23)), so that the cosmological "constant" has here a new value, i.e. it is a variable.

Consequently, by the considerations of this chapter, we can generally say that the variable vacuum energy density is represented correspondingly by a cosmological "constant" and a dark energy. More generally, we treat the vacuum energy density in Sec. 6.

Using the data of Tables III to V, all results, derived in this Sec. 3.2.5 for the universe, are also valid for the anti-universe.

4 The heavy neutrinos

4.1 The rest energy and the number density of the heavy neutrinos

In Refs. [1-3], we have found arguments for the existence of a heavy dark energy neutrino $\tilde{\nu}_\Lambda$, a heavy dark matter neutrino $\tilde{\nu}_{\mathrm{dm}}$, a heavy baryon (ionizable matter) neutrino $\tilde{\nu}_{\mathrm{b}}$ and a heavy "CMB" (photon) neutrino $\tilde{\nu}_{\mathrm{CMB}}$ with the rest energy $E_0(\tilde{\nu}_\Lambda)$, $E_0(\tilde{\nu}_{\mathrm{dm}})$, $E_0(\tilde{\nu}_{\mathrm{b}})$ and $E_0(\tilde{\nu}_{\mathrm{CMB}})$ as well as the number density $n(\tilde{\nu}_\Lambda)$, $n(\tilde{\nu}_{\mathrm{dm}})$, $n(\tilde{\nu}_{\mathrm{b}})$ and $n(\tilde{\nu}_{\mathrm{CMB}})$, respectively. These

heavy neutrinos could be identical with the long-sought, right-handing neutrinos [19].

These arguments are based on the fact, that light neutrino density parameters, multiplied by the products of the different ratios of the relativistic energy to the rest energy of the SUSY GUT particles (X and Y gauge bosons as well as magnetic monopoles), yield the particle-defined, (present), cosmological parameter values (see Sec. 4.2 and Table V), which determine the history and the future of the universe.

In this chapter, for these heavy neutrinos, according to Refs. [1-3], we can so derive their rest energies and number densities via Eqs. (3.34) and (3.44) as well as (3.72) to (3.75), forming the ratios

$$r_1 = \frac{E(\mathrm{M})}{E_0(\mathrm{M})} = \frac{E_{\mathrm{Pl}}}{E_0(\mathrm{M})}, \tag{4.1}$$

$$r_2 = \frac{E(\mathrm{X})}{E_0(\mathrm{X})} = \frac{\frac{1}{2}E_0(\mathrm{M})}{E_0(\mathrm{X})}, \tag{4.2}$$

$$r_3 = \frac{E(\mathrm{Y})}{E_0(\mathrm{Y})} = \frac{\frac{1}{2}E_0(\mathrm{M})}{E_0(\mathrm{Y})}, \tag{4.3}$$

$$r_4 = \frac{E(\mathrm{X}) + E(\mathrm{Y})}{E_0(\mathrm{Y})} = \frac{E_0(\mathrm{M})}{E_0(\mathrm{Y})} \tag{4.4}$$

and

$$r_5 = \left(r_2 \frac{1}{r_4}\right)^{1/2} = \left(\frac{E(\mathrm{X})}{E_0(\mathrm{X})} \frac{E_0(\mathrm{Y})}{E(\mathrm{X}) + E(\mathrm{Y})}\right)^{1/2} = \frac{1}{\sqrt{2}}. \tag{4.5}$$

Then, using the arguments of Refs. [1-3], we find their rest energy and number density as follows:

$$E_0(\tilde{\nu}_\Lambda) = r_1 \cdot r_4 \cdot \sum_i E_0(\nu_i) + r_1 \cdot r_3 \cdot E_0(\nu_\mu) =$$

$$= \sum_i E_0(\nu_i) \frac{E_{\mathrm{Pl}}}{E_0(\mathrm{Y})} + E_0(\nu_\mu) \frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{Y})} = 29.3^{+2.4}_{-2.3}\,\mathrm{eV} \tag{4.6}$$

and

$$n(\widetilde{\nu}_\Lambda) = \frac{\Omega_\Lambda\, \rho_{0C} c^2}{E_0(\widetilde{\nu}_\Lambda)} = 112\ \mathrm{cm}^{-3}, \tag{4.7}$$

$$E_0(\widetilde{\nu}_{\mathrm{dm}}) = r_1 \cdot r_2 \cdot E_0(\nu_\tau) + \sum_i E_0(\nu_i) =$$
$$= E_0(\nu_\tau)\frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{X})} + \sum_i E_0(\nu_i) = 11.31^{+0.94}_{-0.87}\ \mathrm{eV} \tag{4.8}$$

and

$$n(\widetilde{\nu}_{\mathrm{dm}}) = \frac{\Omega_{\mathrm{dm}}\, \rho_{0C} c^2}{E_0(\widetilde{\nu}_{\mathrm{dm}})} = 112\ \mathrm{cm}^{-3}, \tag{4.9}$$

$$E_0(\widetilde{\nu}_{\mathrm{b}}) = r_1 \cdot r_2 \cdot E_0(\nu_\mu) + \sum_i E_0(\nu_i) =$$
$$= E_0(\nu_\mu)\frac{E_{\mathrm{Pl}}}{2E_0(\mathrm{X})} + \sum_i E_0(\nu_i) = 2.08. \pm 0.16\ \mathrm{eV} \tag{4.10}$$

and

$$n(\widetilde{\nu}_{\mathrm{b}}) = \frac{\Omega_{\mathrm{b}}\, \rho_{0C} c^2}{E_0(\widetilde{\nu}_{\mathrm{b}})} = 112\ \mathrm{cm}^{-3} \tag{4.11}$$

as well as

$$E_0(\widetilde{\nu}_{\mathrm{CMB}}) = E_0(\nu_{\mathrm{e}})\, r_1 r_2 r_5 = E_0(\nu_{\mathrm{e}})\frac{E_{\mathrm{Pl}}}{2\sqrt{2}\, E_0(\mathrm{X})} = 0.2564\ \mathrm{eV}. \tag{4.12}$$

Since the light neutrino density parameters (see Eq. (3.25)) were derived by aid of the present number density of the light neutrinos, the number density of the heavy neutrinos (see (4.7), (4.9) and (4.11)) must be identical with the number density of the light neutrinos (3.24) if we consider the derivation method of the estimated, particle-defined (present), cosmological parameter values in Table V according to Refs. [1-3].

Therefore, we assume also $n(\widetilde{\nu}_{\mathrm{CBM}}) = 112\,\mathrm{cm}^{-3}$ (see also Refs. [1-3]), so that we get

$$\Omega_{\tilde{\nu}}(\tilde{\nu}_{\mathrm{CMB}}) = \frac{n(\tilde{\nu}_{\mathrm{CMB}})\, E_0(\tilde{\nu}_{\mathrm{CMB}})}{\rho_{0\mathrm{C}}\, c^2} = 6.02 \cdot 10^{-3}\,. \tag{4.13}$$

Using the data of Tables III to V, all results, derived in this Sec. 4.1 for the universe, are also valid for the anti-universe.

4.2 The derivation of the particle-defined, (present), cosmological parameter values via the rest energy of the heavy neutrinos

According to Refs. [1-3], via $\Omega_\gamma = 5.46 \cdot 10^{-5}$ (see Table I) and Eq. (3.25), by the general transformation $E_0(\nu_i) \to E_0(\tilde{\nu})$, we find the particle-defined, (present) cosmological parameter values as a function of the rest energy of the heavy neutrinos in agreement with Table V, i.e. we have

$$\Omega_\Lambda = 2.347 \cdot 10^{-2}\, \frac{E_0(\tilde{\nu}_\Lambda)}{\mathrm{eV}} = 0.687\,, \tag{4.14}$$

$$\Omega_{\mathrm{dm}} = 2.347 \cdot 10^{-2}\, \frac{E_0(\tilde{\nu}_{\mathrm{dm}})}{\mathrm{eV}} = 0.265\,, \tag{4.15}$$

$$\Omega_{\mathrm{b}} = 2.347 \cdot 10^{-2}\, \frac{E_0(\tilde{\nu}_{\mathrm{b}})}{\mathrm{eV}} = 0.0488 \tag{4.16}$$

and

$$\Omega_{\tilde{\nu}}(\hat{\nu}_{\mathrm{CMB}}) = 2.347 \cdot 10^{-2}\, \frac{E_0(\tilde{\nu}_{\mathrm{CMB}})}{\mathrm{eV}} = 6.02 \cdot 10^{-3}\,. \tag{4.17}$$

By the new thermal equilibrium, because of Eq. (4.12), this heavy "CMB" photon neutrino could be responsible for the decoupling (dec) of the photons [1-3, 5], since the redshift condition

$$1 + z_{\mathrm{dec}} = E_0(\tilde{\nu}_{\mathrm{CMB}})/kT_0 = 1091.7 \tag{4.18}$$

yields the redshift of the photon decoupling to

$$z_{\mathrm{dec}} = 1090.7 \tag{4.19}$$

and this value (4.19) agrees well with the corresponding values $z_{\mathrm{dec}} = 1091 \pm 1$ (see Ref. [9]), $z_{\mathrm{dec}} = 1090.2 \pm 0.7$ (see Ref. [6]) and $z_{\mathrm{dec}} = 1090.43 \pm 0.54$ (see Ref. [11]).

Using the data of Tables III to V, all results, derived in this Sec. 4.2 for the universe, are also valid for the anti-universe.

5 The sterile neutrinos

5.1 The rest energy and the number density of the sterile neutrinos

In Refs. [1-3], we have found arguments for the existence of a sterile dark energy neutrino $\hat{\nu}_\Lambda$, a sterile dark matter neutrino $\hat{\nu}_{\mathrm{dm}}$, a sterile baryon (ionizable matter) neutrino $\hat{\nu}_{\mathrm{b}}$ and a sterile "CMB" (photon) neutrino $\hat{\nu}_{\mathrm{CMB}}$ with the rest energy $E_0(\hat{\nu}_\Lambda)$, $E_0(\hat{\nu}_{\mathrm{dm}})$, $E_0(\hat{\nu}_{\mathrm{b}})$ and $E_0(\hat{\nu}_{\mathrm{CMB}})$ as well as the number density $n(\hat{\nu}_\Lambda)$, $n(\hat{\nu}_{\mathrm{dm}})$, $n(\hat{\nu}_{\mathrm{b}})$ and $n(\hat{\nu}_{\mathrm{CMB}})$, respectively.

These arguments are based on the reliable fact that in Ref. [20] a weak unidentified X-ray line with the energy $E = (3.56 \pm 0.03)\,\mathrm{keV}$ and the photon flux $\Phi = (4.0^{+0.8}_{-0.8}[^{+1.8}_{-1.2}]) \cdot 10^{-6}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ (at 68% [90%] confidence levels) was detected in a stacked $XMM - Newton$ spectrum of 73 galaxy clusters (spanning a redshift range $0.01 - 0.35$).

For this line, in Ref. [20], a dark matter interpretation was assumed via the photon decay of the sterile neutrinos (long-sought dark matter particle candidates). This assumption is reasonable because they decay via the gravitation in one photon with the energy of their half rest energy and in one sterile neutrino relic photon with the energy of their half rest energy as well as possess a lifetime larger than the age of the universe [1-3].

In this chapter, we treat the rest energy and the number density of these sterile neutrinos (see above).

By the decay properties of the sterile neutrinos, via the observations of Ref. [20], we can calculate the rest energy of the sterile dark matter neutrinos to

$$E_0(\hat{\nu}_{\mathrm{dm}}) = 2E = 7120\ \mathrm{eV}, \tag{5.1}$$

so that by $E_0(\tilde{\nu}_{\mathrm{dm}}) = 11.31\,\mathrm{eV}$ (see Eq. (4.8)) we can determine the ratio

$$\frac{E_0(\hat{\nu}_{\mathrm{dm}})}{E_0(\tilde{\nu}_{\mathrm{dm}})} \cong 629.5\,. \tag{5.2}$$

In Refs. [1-4], because of the constant ratio (5.2), we have identified the sterile neutrinos as excited or extremely relativistic state of the heavy neutrinos (see also Eq. (5.33)). Therefore, in analogy to the heavy dark matter neutrinos, for the number density of the sterile dark matter neutrinos, we assume

$$n(\hat{\nu}_{\mathrm{dm}}) = \frac{\Omega_{\mathrm{dm}}\rho_{0\mathrm{C}}c^2}{E_0(\hat{\nu}_{\mathrm{dm}})} = 0.178\ \mathrm{cm}^{-3}\,, \tag{5.3}$$

so that by $n(\tilde{\nu}_{\mathrm{dm}}) = 112\ \mathrm{cm}^{-3}$ (see Eq. (4.9)) we get the ratio

$$\frac{n(\tilde{\nu}_{\mathrm{dm}})}{n(\hat{\nu}_{\mathrm{dm}})} \cong 629.2\,. \tag{5.4}$$

Because all heavy neutrinos possess the same number density (see Sec. 4.1), we assume also this fact for all sterile neutrinos [1-3], so that we have also for the remaining, sterile neutrinos as number density

$$n(\hat{\nu}_{\Lambda}) = 0.178\ \mathrm{cm}^{-3}\,, \tag{5.5}$$

$$n(\hat{\nu}_{\mathrm{b}}) = 0.178\ \mathrm{cm}^{-3}\,, \tag{5.6}$$

and

$$n(\hat{\nu}_{\mathrm{CMB}}) = 0.178\ \mathrm{cm}^{-3}\,, \tag{5.7}$$

i.e. for $j = \Lambda, \mathrm{dm}, \mathrm{b}, \mathrm{CMB}$ by Eqs. (5.2) and (5.4) we can generally introduce

$$\frac{E_0(\hat{\nu}_j)}{E_0(\tilde{\nu}_j)} = \frac{n(\tilde{\nu}_j)}{n(\hat{\nu}_j)} \cong 629.2\,. \tag{5.8}$$

Consequently, by the expression (5.8), we can calculate the rest energies of the remaining, sterile neutrinos because the rest energies of the corresponding, heavy neutrinos are known (see Sec. 4.1), i.e. we obtain

$$E_0(\hat{\nu}_{\Lambda}) = \frac{n(\tilde{\nu}_{\Lambda})}{n(\hat{\nu}_{\Lambda})} E_0(\tilde{\nu}_{\Lambda}) = 18436\ \mathrm{eV}\,, \tag{5.9}$$

$$E_0(\hat{\nu}_{\mathrm{b}}) = \frac{n(\tilde{\nu}_{\mathrm{b}})}{n(\hat{\nu}_{\mathrm{b}})} E_0(\tilde{\nu}_{\mathrm{b}}) = 1309\ \mathrm{eV} \tag{5.10}$$

and

$$E_0(\hat{\nu}_{\mathrm{CMB}}) = \frac{n(\tilde{\nu}_{\mathrm{CMB}})}{n(\hat{\nu}_{\mathrm{CMB}})} E_0(\tilde{\nu}_{\mathrm{CMB}}) = 161.33\ \mathrm{eV}. \tag{5.11}$$

In Refs. [1-3], the fluxes $\Phi(\hat{\nu}_{\mathrm{dm}})$ and $\Phi(\hat{\nu}_{\Lambda})$, which describe correspondingly the photon flux of these sterile neutrinos analogous to the X-ray line with $E = (3.56 \pm 0.03)\,\mathrm{keV}$ and $\Phi = (4.0^{+0.8}_{-0.8}[^{+1.8}_{-1.2}]) \cdot 10^{-6}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ in Ref. [20], were correctly found to

$$\Phi(\hat{\nu}_{\mathrm{dm}}) = \frac{\Omega_{\mathrm{b}} \times (1+z_{\mathrm{MNA}})^2 + \Omega_{\Lambda}/(1+z_{\mathrm{MNA}})}{(\Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}})(1+z_{\mathrm{MNA}})^2 + \Omega_{\Lambda}/(1+z_{\mathrm{MNA}})} \times$$
$$\times \frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0}\, c\, n(\tilde{\nu}_{\mathrm{dm}}) = 4.1 \cdot 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \tag{5.12}$$

and

$$\Phi(\hat{\nu}_{\Lambda}) = \frac{\Omega_{\Lambda}/(1+z_{\mathrm{MNA}})}{(\Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}})(1+z_{\mathrm{MNA}})^2 + \Omega_{\Lambda}/(1+z_{\mathrm{MNA}})} \times$$
$$\times \frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0}\, c\, n(\tilde{\nu}_{\Lambda}) = 1.59 \cdot 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}, \tag{5.13}$$

where according to Table I the particle density parameters are given by $\Omega_{\Lambda} = 0.685$, $\Omega_{\mathrm{dm}} = 0.265$ and $\Omega_{\mathrm{b}} = 0.0499$, whereas the two quantities $1 + z_{\mathrm{MNA}} = 2.77$ and $(R_{\mathrm{earth}} + R_{\mathrm{sun}})/R_0 = 5.105 \cdot 10^{-18}$ are derived in Refs. [1-3] [or by Eqs. (7.18) and (7.34), respectively). The corresponding number densities $n(\tilde{\nu})$ of the heavy neutrinos were defined in Sec. 4.1.

In contrast to $\Phi(\hat{\nu}_{\mathrm{dm}})$ and $\Phi(\hat{\nu}_{\Lambda})$, the fluxes $\Phi(\hat{\nu}_{\mathrm{b}})$ and $\Phi(\hat{\nu}_{\mathrm{CMB}})$, estimated in Refs. [1-3], are incorrect. Therefore, in this work, these fluxes $\Phi(\hat{\nu}_{\mathrm{b}})$ and $\Phi(\hat{\nu}_{\mathrm{CMB}})$ are again derived on the same way as $\Phi(\hat{\nu}_{\mathrm{dm}})$ and $\Phi(\hat{\nu}_{\Lambda})$ in Refs. [1-3], whereat we find

$$\Phi(\hat{\nu}_{\mathrm{b}}) = \frac{(\Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}})(1+z_{\mathrm{MNA}})^2 + \Omega_{\Lambda}/(1+z_{\mathrm{MNA}})}{\Omega_{\mathrm{dm}}(1+z_{\mathrm{MNA}})^2} \times$$
$$\times \frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0}\, c\, n(\tilde{\nu}_{\mathrm{b}}) = 2.25 \cdot 10^{-5}\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \tag{5.14}$$

and

$$\Phi(\hat{\nu}_{\mathrm{CMB}}) = \frac{(\Omega_{\mathrm{dm}} + \Omega_{\mathrm{b}})(1 + z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1 + z_{\mathrm{MNA}})}{\Omega_\Lambda/(1 + z_{\mathrm{MNA}})} \times$$

$$\times \frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0}\, c\, n(\tilde{\nu}_{\mathrm{CMB}}) = 1.85 \cdot 10^{-4}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}. \tag{5.15}$$

Using Eqs. (5.1) as well as (5.9) to (5.11), we get, for example, the ratios

$$\frac{E_0(\hat{\nu}_\Lambda)}{E_0(\hat{\nu}_{\mathrm{dm}})} = \frac{\Omega_\Lambda}{\Omega_{\mathrm{dm}}} \cong 2.59\,, \tag{5.16}$$

$$\frac{E_0(\hat{\nu}_\Lambda)}{E_0(\hat{\nu}_{\mathrm{b}})} = \frac{\Omega_\Lambda}{\Omega_{\mathrm{b}}} \cong 14 \tag{5.17}$$

and

$$\frac{E_0(\hat{\nu}_\Lambda)}{E_0(\hat{\nu}_{\mathrm{CMB}})} = \frac{\Omega_\Lambda}{\Omega_{\tilde{\nu}}(\hat{\nu}_{\mathrm{CMB}})} \cong 114\,. \tag{5.18}$$

The expressions (5.12) to (5.15), multiplied by the corresponding, half rest energies of the sterile neutrinos (see Eqs. (5.1) as well as (5.9) to (5.11)), yield the same energy fluxes for the photons, so that we can again form the ratios

$$\frac{E_0(\hat{\nu}_\Lambda)}{E_0(\hat{\nu}_{\mathrm{dm}})} = \frac{\Phi(\hat{\nu}_{\mathrm{dm}})}{\Phi(\hat{\nu}_\Lambda)} \cong 2.59\,, \tag{5.19}$$

$$\frac{E_0(\hat{\nu}_\Lambda)}{E_0(\hat{\nu}_{\mathrm{b}})} = \frac{\Phi(\hat{\nu}_{\mathrm{b}})}{\Phi(\hat{\nu}_\Lambda)} \cong 14 \tag{5.20}$$

and

$$\frac{E_0(\hat{\nu}_\Lambda)}{E_0(\hat{\nu}_{\mathrm{CMB}})} = \frac{\Phi(\hat{\nu}_{\mathrm{CMB}})}{\Phi(\hat{\nu}_\Lambda)} \cong 114\,. \tag{5.21}$$

The excellent agreement of the corresponding expressions (5.16) to (5.21) supports the correctness of the estimations (5.12) to (5.15).

Using the ratios (5.16) and (5.17) for the expressions (5.12) to (5.15), we can obtain the fluxes (5.12) to (5.15) in dependence on the rest energies of all sterile neutrinos except the sterile neutrino $\hat{\nu}_{\mathrm{CMB}}$, so that we can correspondingly assume a joint origin of the fluxes (5.12) to (5.15) by these sterile neutrinos except the sterile neutrino $\hat{\nu}_{\mathrm{CMB}}$.

Therefore, in the present, the dark matter or dark energy, represented by the fluxes (5.12) or (5.13), are described by the sterile neutrinos except the sterile neutrino $\hat{\nu}_{\mathrm{CMB}}$, which is coupled with the heavy neutrino $\tilde{\nu}_{\mathrm{CMB}}$ and is so responsible for the photon decoupling (see Sec. 4.2), i.e. the three sterile neutrinos $\hat{\nu}_{\Lambda}$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ define so a joint origin of the dark matter and dark energy [1-3].

Now, we must explain only still the meaning of the flux $\Phi(\hat{\nu}_{\mathrm{b}})$ in Eq. (5.14). In this case, we assume that the flux $\Phi(\hat{\nu}_{\mathrm{b}})$ is equivalent to the flux $\Phi(\tilde{\nu}_{\mathrm{b}})$, which corresponds to the additional contribution of all heavy neutrinos to the present, ionizable matter $\Omega_{\mathrm{b}} = \overline{\Omega}_{\mathrm{b}} + \Omega_{\nu}$ [1-3] because of all light neutrinos (Ω_{ν}), since in Refs. [1, 2] we have also explained the sterile neutrinos as the excited or extremely relativistic state of the corresponding heavy neutrinos (see also Eq. (5.33)), whereat we have shown also the transformations of various types of all neutrinos into one another (see Refs. [1, 2].

For this goal, using the data of these neutrino types in Secs. 3.1 and 4, we estimate their neutrino energy density ε_{ν} [1, 2] to

$$\varepsilon_{\nu} = \Omega_{\nu}\rho_{0\mathrm{C}}c^2 = n_0(\nu_i)\sum_i E_0(\nu_i) = n(\tilde{\nu}_{\mathrm{b}})\sum_i E_0(\nu_i) =$$
$$= n(\tilde{\nu}_{\mathrm{dm}})\sum_i E_0(\nu_i) = n(\tilde{\nu}_{\Lambda})\sum_i E_0(\nu_i) = 6.69\ \mathrm{eV\,cm^{-3}}. \qquad (5.22)$$

Using from Eq. (5.22) the four different unique terms with the joint factor $\sum_i E_0(\nu_i) = 5.97\cdot 10^{-2}\,\mathrm{eV}$ (see Eq. (3.11)), for this contribution of these neutrinos to the ionisable matter Ω_{b}, we find for their total neutrino energy density [1, 2] to

$$4\,\varepsilon_{\nu} \approx 26.8\ \mathrm{eV\,cm^{-3}}. \qquad (5.23)$$

Using $\Omega_{\mathrm{b}} = 0.0487$ (see Tables I and V), the ionizable matter is defined by the total baryon energy density [1, 2, 6, 7] to

$$\varepsilon_{\mathrm{b}} = \Omega_{\mathrm{b}}\rho_{0\mathrm{C}}c^2 = n(\tilde{\nu}_{\mathrm{b}})\,E_0(\tilde{\nu}_{\mathrm{b}}) = n_{\mathrm{b}}E_0(\mathrm{p}) \approx 232.9\ \mathrm{eV\,cm^{-3}}, \qquad (5.24)$$

where $n_{\mathrm{b}} = 2.482\cdot 10^{-7}\,\mathrm{cm^{-3}}$ [6] and $E_0(\mathrm{p}) = 9.38272\cdot 10^{8}\,\mathrm{eV}$ [6, 9] are the baryon number density and the proton rest energy, respectively.

Then, the expected flux $\Phi(\tilde{\nu}_{\mathrm{b}})$, can be evaluated to

$$\Phi(\tilde{\nu}_{\mathrm{b}}) = \frac{E_0(\tilde{\nu}_{\mathrm{b}})}{E_0(\mathrm{p})} \frac{(\varepsilon_{\mathrm{b}} + 12\,\varepsilon_\nu)}{E_0(\mathrm{p})} c = 2.22 \cdot 10^{-5}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,, \tag{5.25}$$

where the quantity $12\,\varepsilon_\nu$ is defined via the expression (5.23), multiplied by the number 3 of the sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ (see above), i.e. $\varepsilon_{\mathrm{b}} + 12\,\varepsilon_\nu \approx 313.3\,\mathrm{eV\,cm}^{-3}$ represents the total energy density of the heavy neutrinos. Indeed, the agreement of the fluxes (5.14) and (5.25) confirms our assumption.

Using the data of Tables III to V, all results, derived in this Sec. 5.1 for the universe, are also valid for the anti-universe.

5.2 The derivation of the particle-defined, (present) cosmological parameter values via the rest energy of the sterile neutrinos

According to Refs. [1-3], via $\Omega_\gamma = 5.46 \cdot 10^{-5}$ (see Table I) and Eq. (3.25), by the general transformation $E_0(\nu_i) \to E_0(\tilde{\nu}_J) = \{n(\hat{\nu}_j)/n(\tilde{\nu}_j)\} E_0(\hat{\nu}_j)$ [see Eq. (5.8)], we have also calculated particle-defined, (present), cosmological parameter values (see also Eqs. (4.14) to (4.17)) as a function of the rest energy of the sterile neutrinos (see Eqs. (5.1) as well as (5.9) to (5.11)) in agreement with Table V, i.e. we have

$$\Omega_\Lambda = 3.730 \cdot 10^{-5} \frac{E_0(\hat{\nu}_\Lambda)}{\mathrm{eV}} = 0.687\,, \tag{5.26}$$

$$\Omega_{\mathrm{dm}} = 3.730 \cdot 10^{-5} \frac{E_0(\hat{\nu}_{\mathrm{dm}})}{\mathrm{eV}} = 0.265\,, \tag{5.27}$$

$$\Omega_{\mathrm{b}} = 3.730 \cdot 10^{-5} \frac{E_0(\hat{\nu}_{\mathrm{b}})}{\mathrm{eV}} = 0.0488 \tag{5.28}$$

and

$$\Omega_{\tilde{\nu}}(\tilde{\nu}_{\mathrm{CMB}}) = 3.730 \cdot 10^{-5} \frac{E_0(\hat{\nu}_{\mathrm{CMB}})}{\mathrm{eV}} = 6.02 \cdot 10^{-3}\,. \tag{5.29}$$

Using the data of Tables III to V, all results, derived in this Sec. 5.2 for the universe, are also valid for the anti-universe.

5.3 A semi-empirical explanation of the sterile neutrino calculations

We assume that (because of Sec. 5.2) Eq. (5.8) must be attributable to the light and the heavy neutrinos.

If we use $E_0(\tilde{\nu}_\Lambda) = 29.3\,\mathrm{eV}$ (see Eq. (4.6)) and $E_0(\nu_\tau) = 0.0493\,\mathrm{eV}$ (see Eq. (3.9)) as well as $E_0(\tilde{\nu}_\mathrm{b}) = 2.08\,\mathrm{eV}$ (see Eq. (4.10) and $\sum_i E_0(\nu_i) = 0.0597\,\mathrm{eV}$ (see Eq. (3.11)), we find semi-empirically the connection

$$\frac{E_0(\hat{\nu}_j)}{E_0(\tilde{\nu}_j)} = \frac{n(\tilde{\nu}_j)}{n(\hat{\nu}_j)} = \frac{E_0(\tilde{\nu}_\Lambda)}{E_0(\nu_\tau)} + \frac{E_0(\tilde{\nu}_\mathrm{b})}{\sum_i E_0(\nu_i)} = 629.2\,, \tag{5.30}$$

where we have again $j = \Lambda, \mathrm{dm}, \mathrm{b}, \mathrm{CMB}$.

This connection is useful because the number densities of the sterile neutrinos (see Sec. 5.1) are generally defined by

$$n(\hat{\nu}_j) = \frac{E_0(\tilde{\nu}_j)}{E_0(\hat{\nu}_j)}\, n(\tilde{\nu}_j) = 0.178\ \mathrm{cm}^{-3}\,, \tag{5.31}$$

since the number densities $n(\tilde{\nu}_j) = 112\,\mathrm{cm}^{-3}$ of the heavy neutrinos are known (see Sec. 4.1). Analogously, the rest energies of the sterile neutrinos (see Sec. 5.1) are generally given by

$$E_0(\hat{\nu}_j) = \frac{n(\tilde{\nu}_j)}{n(\hat{\nu}_j)}\, E_0(\tilde{\nu}_j)\,, \tag{5.32}$$

since the rest energies $E_0(\tilde{\nu}_j)$ of the heavy neutrinos are also known (see Sec. 4.1).

Because of the result (5.30), the sterile neutrino types can be interpreted as the extremely relativistic or excited state of the corresponding heavy neutrinos, i.e. the heavy neutrinos must possess the same velocities

$$v(\tilde{\nu}_j) = (1 - [E_0(\tilde{\nu}_j)/E_0(\hat{\nu}_j)]^2)^{1/2} c = 0.99999874\, c\,. \tag{5.33}$$

Consequently, our interpretations in Secs. 5.1 and 5.2 are independent on the observation of the X-ray line $E = (3.56 \pm 0.03)\,\mathrm{keV}$.

Thus, the description of the present, dark matter and dark energy is unique by the three sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_\mathrm{dm}$ and $\hat{\nu}_\mathrm{b}$, which form together the joint

origin of the present, dark matter and dark energy according to Sec. 5.1, where this assumption was first introduced. A similar assumption was already expressed in Refs. [1-3].

The existence of these sterile neutrinos (for example, as long-sought dark matter particle candidate) was reliable detected via the X-ray line $E = (3.56 \pm 0.03)\,\mathrm{keV}$ because it was observed by several, independent researcher groups [20-25].

Using the data of Tables III to V, all results, derived in this Sec. 5.3 for the universe, are also valid for the anti-universe.

6 A simple solution for the four-dimensional quantum gravity via the vacuum energy density

The considerations of the total chapter 6 are based on Ref. [1]. In this chapter 6, we derive a simple solution for the four-dimensional quantum gravity (quantum cosmology) via the vacuum energy density.

We use the fact that for a particle gas in a cube with rigid walls and the particle density n between the gas pressure P and the mean momentum changing Δp of the particles according to the kinetic theory of gases [10] we have the relation

$$P = \frac{1}{6} n \langle v \, \Delta p \rangle , \tag{6.1}$$

where v is the velocity of the single particle, i.e. the positive momentum changing $\Delta p = 2p$ is the momentum transfer of the single particle on the rigid wall, if p is the perpendicular component of the single particle momentum on the rigid wall, so that we obtain a positive pressure

$$P = \frac{1}{3} n \langle v \, p \rangle . \tag{6.2}$$

However, if the particle itself is connected with the momentum changing and the imaginary wall is not related with the momentum changing, we have

a negative momentum changing $\Delta p = -2p$ [10], i.e. we get a negative pressure

$$P = -\frac{1}{3} n \langle v\, p \rangle , \tag{6.3}$$

For photons, we have $\langle v \rangle = c$ and $\langle p \rangle = E/c$, so that Eq. (6.2) gives

$$P = \frac{1}{3} n \langle E \rangle = \frac{1}{3} \rho_\gamma c^2 , \tag{6.4}$$

whereas Eq. (6.3) yields

$$P = -\frac{1}{3} n \langle E \rangle = -\frac{1}{3} \rho_\gamma c^2 , \tag{6.5}$$

According to Eq. (6.1), at the scale factor R, the positive, imaginary wall loading is $\langle \Delta p \rangle = 2E/c$ as the mean momentum changing by the photons.

In contrast to Eq. (6.4), the negative pressure (6.5) means that according to Eq. (6.1) the imaginary wall loading at the scale factor R is zero, since the photon itself is related to the negative momentum changing $\langle \Delta p \rangle = -2E/c$.

The negative pressure (6.5) must be connected with the vacuum energy density (vacuum energy) according to quantum field theory, which yields

$$\rho_{\text{vac}} c^2 = -P = \frac{1}{3} \rho_\gamma c^2 . \tag{6.6}$$

In the cosmological models, this vacuum energy density, which is connected with the dark energy, is represented by the so-called cosmological "constant" [1-5, 7]. Therefore, Eqs. (6.5) and (6.6) mean a new, additional possibility at the description of the massless and the massive universes by photons if they are in the new thermal equilibrium with relativistic particles or antiparticles (see Sec. I or the paragraph after Eq. (3.36) in Sec. 3.2.1).

Thus, the considerations, which are performed in the following chapters 6.1 to 6.5, are based on Eq. (6.6). This fact is not explained always because it is clear without a reference at the corresponding text positions.

6.1 The vacuum energy at $R = R_{\mathrm{Pl}}$ via a modified, quantum-statistical expression for the photon energy density

At $R = R_{\mathrm{Pl}}$, for the derivation of the vacuum energy density, by photons $E = kT_{\mathrm{Pl}}$, according to the negative pressure (see Eq. (6.6)), we must interpret the expression

$$\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2 = \rho_{\mathrm{vac}}(T_{\mathrm{Pl}})c^2 = \frac{1}{3}\rho_\gamma(T_{\mathrm{Pl}})c^2 . \tag{6.7}$$

Using the quantum-statistical photon energy density (3.13) for $\rho_\gamma(T_{\mathrm{Pl}})c^2$ at the real radiation field temperature T_{Pl} (see Sec. I), we suggest

$$\rho_{\mathrm{vac}}(T_{\mathrm{Pl}})c^2 \propto \tfrac{1}{3}\frac{\pi^2}{15}\frac{(kT_{\mathrm{Pl}})^4}{(\hbar c)^3} . \tag{6.8}$$

Then, via Eqs. (3.51) and (3.64), we have the additional condition

$$\widetilde{R} = R_{\mathrm{Pl}} = (2N(T)\,\Omega_\gamma)^{1/4}(R_{\mathrm{Pl}}\,c\,t_{\mathrm{Pl}})^{1/2} , \tag{6.9}$$

so that by Eq. (6.9) we can write

$$\frac{R_{\mathrm{Pl}}}{(2\Omega_\gamma)^{1/4}} = [N(T)]^{1/4} R_{\mathrm{Pl}} , \tag{6.10}$$

i.e. by $R_{\mathrm{Pl}} = \hbar c/kT_{\mathrm{Pl}}$ we find

$$(2\Omega_\gamma)^{1/4} T_{\mathrm{Pl}} = \frac{T_{\mathrm{Pl}}}{[N(T)]^{1/4}} . \tag{6.11}$$

Consequently, because of Eq. (6.11), we assume that the right term of Eq. (6.8) must be multiplied by $1/N(T)$ or $2\Omega_\gamma$ ($\Omega_\gamma = 5.46\cdot 10^{-5}$ see Table I) to get an equation, so that we find

$$\rho_{\mathrm{vac}}(T_{\mathrm{Pl}})c^2 = \tfrac{1}{3}\frac{\pi^2}{15}\frac{(kT_{\mathrm{Pl}})^4}{(\hbar c)^3}\frac{1}{N(T)} . \tag{6.12}$$

and

$$\rho_{\mathrm{vac}}(T_{\mathrm{Pl}})c^2 = \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{(kT_{\mathrm{Pl}})^4}{(\hbar c)^3} = (6.93 \pm 0.24)\cdot 10^{121}\ \mathrm{eV/cm^3} \tag{6.13}$$

in excellent agreement with the value of Eq. (3.70), i.e. by this result the assumptions of Eqs. (6.5) and (6.6) are confirmed for photons with the thermal energy $E = kT_{\mathrm{Pl}}$ and $1/N(T) = 2\Omega_\gamma$.

This excellent agreement confirms also the correctness of the estimations of the rest energy of the neutrinos (see Eqs. (3.8) to (3.11)) and the supersymmetric grand unification particles [X and Y gauge bosons (see Eq. (3.34)) as well as magnetic monopoles (see Eq. (3.44))].

Using Eqs. (7.4) and (6.13) as well as $2\Omega_\gamma = 1/N(T)$ [see above] at $R = R_{\mathrm{Pl}}$, we can also write

$$\rho_{\mathrm{vac}}(T_{\mathrm{Pl}})\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(kT_{\mathrm{Pl}})^4}{(\hbar c)^3} = \frac{1}{3}\frac{\pi^2}{15}\frac{c^7}{G_N^2\hbar}\frac{1}{N(T)}, \tag{6.14}$$

where the factor $c^7/G_N^2\hbar$ is the Planck energy density $\rho_{\mathrm{Pl}}c^2$, which has the value

$$\rho_{\mathrm{Pl}}c^2 = \frac{c^7}{G_N^2\hbar} = 2.89206(69)\cdot 10^{126}\,\mathrm{eV\,cm^{-3}}, \tag{6.15}$$

i.e. the vacuum energy density (6.14) of the massless and massive universe (because of their joint boundary $R = R_{\mathrm{Pl}}$) is smaller than the Planck energy density (6.15) by a factor β of

$$\beta = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} = 2.395\cdot 10^{-5}, \tag{6.16}$$

so that we have

$$\rho_{\mathrm{vac}}(T_{\mathrm{Pl}})\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(kT_{\mathrm{Pl}})^4}{(\hbar c)^3} = \beta\,\frac{c^7}{G_N^2\hbar} = \beta\,\rho_{\mathrm{Pl}}c^2\,. \tag{6.17}$$

Between cosmological "constant" Λ and the vacuum energy density $\rho_{\mathrm{vac}}c^2$, the quantum field theory gives the general connection

$$\Lambda = \frac{8\pi G_N}{c^2}\rho_{\mathrm{vac}} = 8\pi\frac{\hbar c}{E_{\mathrm{Pl}}^2}\rho_{\mathrm{vac}}\,c^2, \tag{6.18}$$

so that by Eqs. (6.14) or (6.17) we get

$$\Lambda = \frac{16}{45}\pi^3\Omega_\gamma \frac{(kT_{\rm Pl})^2}{(\hbar c)^2} = \frac{16}{45}\pi^3\Omega_\gamma \frac{c^3}{G_N\hbar} =$$

$$= \frac{16}{45}\pi^3\Omega_\gamma \frac{1}{R_{\rm Pl}^2} = (2.30 \pm 0.08)\cdot 10^{66}\,{\rm m}^{-2} \tag{6.19}$$

in excellent agreement with the result (3.69).

Generally, using the data of Tables III to V, all results, derived in this Sec. 6.1 for the massless and massive universe, are also valid for the massless and massive anti-universe because of their joint boundary $R = -R_{\rm Pl}$.

6.2 Vacuum energy density directly via gravitation by photons and particles at $R = R_{\rm Pl}$

According to Tables III and IV, for the massless universe, in their real radiation fields (see Ref. [1] and also Sec. I), at $R = R_{\rm Pl}$, the gravitational (gr) potential energy (work) $V_{\rm gr}$ for the massless photons, for which we assume here the thermal energy $kT_{\rm Pl} = E_{\rm Pl}$, yields

$$V_{\rm gr} = \frac{G_N}{c^4}\frac{(kT_{\rm Pl})^2}{R_{\rm Pl}} = \frac{G_N}{c^4}\frac{E_{\rm Pl}^2}{R_{\rm Pl}} = \frac{\hbar c}{R_{\rm Pl}} = \sqrt{\frac{\hbar c^5}{G_N}} = E_{\rm Pl}\,, \tag{6.20}$$

i.e. because of the new thermal equilibrium we can interpret also this potential energy (6.20) as the relativistic energy $E_{\rm Pl}$ of all particles in the massless and massive universe at their joint boundary $R = R_{\rm Pl}$ (energy conservation (see Sec. 8.4)).

Then, Eq. (6.20), divided by the quantum volume $R_{\rm Pl}^3$, yields the Planck energy density

$$\rho_{\rm Pl}c^2 = \frac{E_{\rm Pl}}{R_{\rm Pl}^3} = \frac{V_{\rm gr}}{R_{\rm Pl}^3} = \frac{G_N}{c^4}\frac{(kT_{\rm Pl})^2}{R_{\rm Pl}^4} = \frac{\hbar c}{R_{\rm Pl}^4} = \frac{c^7}{G_N^2\hbar}\,, \tag{6.21}$$

so that via the results (6.14) to (6.17) at the joint boundary $R = R_{\rm Pl}$ the vacuum energy density of the massless and massive universe is also found for particles to

$$\rho_{\text{vac}}(E_{\text{Pl}})c^2 = \rho_{\text{vac}}(T_{\text{Pl}})c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{V_{\text{gr}}}{R_{\text{Pl}}^3} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\rho_{\text{Pl}}c^2 =$$

$$= \frac{1}{3}\frac{\pi^2}{15}\frac{c^7}{G_N^2\hbar}\frac{1}{N(T)} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(E_{\text{Pl}})^4}{(\hbar c)^3} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{(kT_{\text{Pl}})^4}{(\hbar c)^3}. \qquad (6.22)$$

Consequently, the cosmological "constant" is again defined by Eq. (6.19).

Thus, at the joint boundary $R = R_{\text{Pl}}$, the energy $kT_{\text{Pl}} = E_{\text{Pl}}$ has twofold meaning as massless photons or massive particles. This fact characterizes the identity of the end of the massless universe or the beginning of the massive universe.

Generally, using the data of Tables III to V, all results, derived in this Sec. 6.2 for the massless and massive universe, are also valid for the massless and massive anti-universe because of their joint boundary $R = -R_{\text{Pl}}$.

6.3 The quantum gravity as a result of the vacuum energy density directly via the gravitation by photons and particles for $R \le R_{\text{Pl}}$

Taking the data of Tables III and IV as well as the result (6.20), in the massless universe ($R \le R_{\text{Pl}}$) and anti-universe ($-R \le -R_{\text{Pl}}$), which have the distance R' prior to their separation by their expansion in opposite directions (momentums $\pm p$ (see Table IV)), the rest energy of the virtual particle(P)-antiparticle($\overline{\text{P}}$) pairs of the quantum vacuum can be described by the gravitational potential energy (see beginning of Sec. 6.2)

$$V_{\text{gr}}^*(R') = \frac{G_N}{c^4}\frac{(kT_{\text{vac}})(kT_{\text{vac}})}{R'}, \qquad (6.23)$$

if we introduce

$$kT_{\text{vac}} = kT + kT = E_0(\text{P}) + E_0(\overline{\text{P}}) \qquad (6.24)$$

and

$$V_{\text{gr}}^*(R') = kT_{\text{vac}}, \qquad (6.25)$$

so that for $E_0(\text{P}) = E_0(\overline{\text{P}})$ we obtain the Schwarzschild radius (see Sec. 7.1)

$$R' = \frac{G_N}{c^4} kT_{\text{vac}} = \frac{G_N}{c^4}\left[E_0(\text{P}) + E_0(\overline{\text{P}})\right]. \tag{6.26}$$

Because of the very high photon density of these massless universes, after their separation into the massless universe and anti-universe, the description of these rest energies is possible by

$$kT = E_0(\text{P}) \tag{6.27}$$

and

$$kT = E_0(\overline{\text{P}}), \tag{6.28}$$

where kT are correspondingly the thermal energy of the photons in the separated, massless universe and anti-universe. Now, we explain this effect. For this goal, we use the condition (6.24) in the expression (6.23), so that because of $E_0(\text{P}) = E_0(\overline{\text{P}})$ we get

$$\begin{aligned} V^*_{\text{gr}}(R') &= 2E_0(\text{P}) = 2kT = \frac{G_N}{c^4}\frac{(kT)^2}{R'} + \frac{G_N}{c^4}\frac{(kT)(kT)}{R'} + \\ &+ \frac{G_N}{c^4}\frac{(kT)(kT)}{R'} + \frac{G_N}{c^4}\frac{(kT)^2}{R'} = 4V_{\text{gr}}(R') = 4\times\frac{1}{2}kT = \\ &= \frac{G_N}{c^4}\frac{E_0^2(\text{P})}{R'} + \frac{G_N}{c^4}\frac{E_0(\overline{\text{P}})\times E_0(\text{P})}{R'} + \\ &+ \frac{G_N}{c^4}\frac{E_0(\text{P})\times E_0(\overline{\text{P}})}{R'} + \frac{G_N}{c^4}\frac{E_0^2(\overline{\text{P}})}{R'} = 4V_{\text{gr}}(R') = 4\times\frac{1}{2}E_0(\text{P}). \end{aligned} \tag{6.29}$$

Now, we can use the relation $2V^*_{\text{gr}}(R') = 2\times 4V_{\text{gr}}(R') = V^*_{\text{gr}}(R) = 4V_{\text{gr}}(R)$, where at $V^*_{\text{gr}}(R')$ and $4V_{\text{gr}}(R')$ the factor 2 represents together the massless universe and anti-universe, so that we can write

$$V_{\text{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)^2}{R} = \frac{G_N}{c^4}\frac{E_0^2(\text{P})}{R}, \tag{6.30}$$

$$V_{\text{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)(kT)}{R} = \frac{G_N}{c^4}\frac{E_0(\overline{\text{P}})\times E_0(\text{P})}{R}, \tag{6.31}$$

$$V_{\text{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)(kT)}{R} = \frac{G_N}{c^4}\frac{E_0(\text{P})\times E_0(\overline{\text{P}})}{R} \tag{6.32}$$

and

$$V_{\mathrm{gr}}(R) = \frac{G_N}{c^4}\frac{(kT)^2}{R} = \frac{G_N}{c^4}\frac{E_0^2(\overline{\mathrm{P}})}{R}, \tag{6.33}$$

if we introduce $V_{\mathrm{gr}}(R) = kT = E_0(\mathrm{P})$, for example, by Eq. (6.30), i.e. we obtain

$$\frac{R'}{2} = R = \frac{G_N}{c^4}kT = \frac{\hbar c}{E_{\mathrm{Pl}}^2}kT = \frac{G_N}{c^4}E_0(\mathrm{P}). \tag{6.34}$$

Because of $E_0(\mathrm{P}) = E_0(\overline{\mathrm{P}})$, the gravitational potential energies (6.31) to (6.34) are the same for universe and anti-universe in spite of the results in Table IV. The expressions (6.30) and (6.33) can be considered thus as a separation into matter and antimatter by the expansion of the massless universe and anti-universe in opposite directions, whereat their origin is formed by zero-point oscillations (see also Sec. 8.4).

We can assume that the terms (6.30) and (6.33) must be proportional to the vacuum energy density [vacuum energy (dark energy)], which determines the expansion dynamics of the massless universe and anti-universe, where the mixed terms (6.31) and (6.32) must be proportional to the field energy of the quantum vacuum because they disappear by annihilation after an extremely short lifetime, so that their mean energy density must represent this field energy.

Now, we demonstrate this behaviour, for example, via the massless universe. This demonstration is possible because the terms (6.32) and (6.33) of the massless anti-universe possess a completely analogous behaviuor as the corresponding expressions (6.31) and (6.30) of the massless universe.

Then, for the massless universe, we introduce a new energy density via Eqs. (6.30) and (6.31), divided by the quantum volume R^3, i.e. because of Eq. (6.34) we get

$$\rho_{\mathrm{vac}}(R)\,c^2 = \mathrm{const}\frac{\hbar c}{E_{\mathrm{Pl}}^2}\frac{(kT)^2}{R^4} = \mathrm{const}\frac{E_{\mathrm{Pl}}^2}{\hbar c\,R^2} \tag{6.35}$$

and

$$"\rho_{\mathrm{vac}}(R)\,c^2" = \mathrm{const}\frac{\hbar c}{E_{\mathrm{Pl}}^2}\frac{(kT)^2}{R^4} = \mathrm{const}\frac{\overline{E}_{\mathrm{Pl}} \times E_{\mathrm{Pl}}}{\hbar c\,R^2}. \tag{6.36}$$

For $R = R_{\mathrm{Pl}}$, the expression (6.35) must yield the Planck energy density, so that according to the quantum-statistical expressions (6.14) to (6.17) as well as (6.22) we find the vacuum energy density (vacuum energy) of the massless universe by Eq. (6.35), if we assume $\mathrm{const} = 1/N(T)$ and $N(T) = 1/2\Omega_\gamma$ for $R \leq R_{\mathrm{Pl}}$, i.e. we obtain

$$\rho_{\mathrm{vac}}(R)c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{\hbar c}{E_{\mathrm{Pl}}^2}\frac{(kT)^2}{R^4} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\mathrm{Pl}}^2}{\hbar c R^2}. \tag{6.37}$$

Analogous to Eq. (6.37), the mixed expressions (6.36) is also defined by

$$"\rho_{\mathrm{vac}}(R)c^2" = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{\hbar c}{E_{\mathrm{Pl}}^2}\frac{(kT)^2}{R^4} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{\overline{E}_{\mathrm{Pl}} \times E_{\mathrm{Pl}}}{\hbar c R^2}, \tag{6.38}$$

which contains anti-energy as well as energy and disappears therefore after an extremely short lifetime by annihilation, so that it can form a mean energy density. Because of the energy conservation, this mean energy density provides the field energy for the quantum vacuum. Consequently, this field penetrates completely the space-time continuum of the massless universe.

In Eq. (6.37), the limiting values $kT = E_{\mathrm{Pl}}$ and $R = R_{\mathrm{Pl}}$ mean also that all particles have the greatest possible, relativistic particle energy E_{Pl} in the massless and massive universe because of their joint boundary $R = R_{\mathrm{Pl}}$.

The vacuum energy of the massless universe, which is connected with the dark energy is responsible for its expansion dynamics.

Using the connection between cosmological constant and vacuum energy density (see Eq. (6.18)), we obtain the following expression for the cosmological "constant"

$$\Lambda = \frac{16}{45}\pi^3\Omega_\gamma \frac{1}{R^2}. \tag{6.39}$$

For photons, in accordance with $R_{\mathrm{Pl}} = ct_{\mathrm{Pl}}$, we can assume the connection

$$R = ct. \tag{6.40}$$

Because of $kT = E_0(\mathrm{P})$ [see Eq. (6.27)] and $kT = E_{\mathrm{Pl}}^2 R/\hbar c$ (see Eq. (6.34)), we get for the thermal particle number density

$$n_{\mathrm{P}}(R) = \frac{\rho_{\mathrm{vac}}(R)c^2}{kT} = \frac{\rho_{\mathrm{vac}}(R)c^2}{E_0(\mathrm{P})} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R^3}. \tag{6.41}$$

For $kT = E_{\mathrm{Pl}}^2 R/\hbar c$ (see Eq. (6.34)), Eq. (3.21) provides the kinetic energy of the photons to

$$E_{\mathrm{K}}(\gamma) = 2.701178\, kT = 2.701178\, \frac{E_{\mathrm{Pl}}^2}{\hbar \mathrm{c}} R\,, \tag{6.42}$$

so that the thermal photon number density is given by

$$n_\gamma(R) = 2.701178 \frac{\rho_{\mathrm{vac}}(R)c^2}{E_{\mathrm{K}}(\gamma)} = \frac{\rho_{\mathrm{vac}}(R)c^2}{kT} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R^3}\,, \tag{6.43}$$

i.e. we have $n_{\mathrm{P}}(R) = n_\gamma(R)$.

Consequently, in the four-dimensional quantum gravity, the radiation energy density and the vacuum energy density as well as the thermal photon number density and the particle number density are equivalent. Therefore, this simple quantum gravity solves the formation of all known particles $E_0(\mathrm{P})$ via the gravitation (see Eqs. (6.30) and (6.34)), whereat at $R = R_{\mathrm{Pl}}$ these particles have the Planck energy as their greatest possible relativistic energy (see Eq. (6.22)). Thus, this Planck energy (as greatest possible uncertainty of the quantum fluctuation for the distance $R = R_{\mathrm{Pl}}$) is the boundary between the massive universe ($R \geq R_{\mathrm{Pl}}$) and its quantum gravity ($R \leq R_{\mathrm{Pl}}$) for all their particles (energy conservation (see also Sec. 8.4)).

This four-dimensional quantum gravity, which is explicitly determined completely by the expressions (6.34) and (6.37) as well as (6.39) to (6.43), yields uniquely the vacuum energy density (6.37) or the cosmological "constant" (6.39) of the massless universe and anti-universe according to the data of Tables III to V, i.e. we have found a simple quantum-theoretical, four-dimensional (space and time) description for the gravitation and antigravitation.

In Sec. 8, the usefulness of this four-dimensional quantum gravity is shown by its application at the transition from the final state of the massive universes to the big bang as the beginning of the massless universes.

6.4 The vacuum energy density at $R \geq R_{\mathrm{Pl}}$ as quantum gravity via the modified, quantum-statistical expression of photon energy density

In contrast to the old inflation model $R = R_1 \mathrm{e}^{H\Delta t}$ (see Eq. (2.24)), in the new inflation model (see Sec. 3.2.4 and the beginning of Sec. 3.2.5), the inflation is characterized by $R = \widetilde{R}\,\mathrm{e}^{H\Delta t} = \widetilde{R} \times (1 + z_{\mathrm{M}})$ according to Eqs. (3.58) and (3.59), i.e. because of the redshift condition $1 + z$ instead of the integration constant R_1 we have assumed the transformations

$$R_1 \to \widetilde{R} = \widetilde{R}_1(z) = \frac{\widetilde{R}_0}{1+z} \tag{6.44}$$

and

$$R_1 \to \widetilde{R} \times (1 + z_{\mathrm{M}}) = R = \widetilde{R}_1(z) \times (1 + z_{\mathrm{M}}) =$$
$$= \frac{\widetilde{R}_0}{1+z} \times (1 + z_{\mathrm{M}}) = \frac{R_0}{1+z}\,. \tag{6.45}$$

Consequently, these transformations mean that via the total, early universe (6.44) the new inflation model leads to the later universe (6.45). For $z = 0$, we have the "real" integration constants

$$R_1 \to \widetilde{R}_1(z = 0) = \widetilde{R}_0 \tag{6.46}$$

and

$$R_1 \to \widetilde{R}_1(z = 0) \times (1 + z_{\mathrm{M}}) = \widetilde{R}_0 \times (1 + z_{\mathrm{M}}) = R_0\,, \tag{6.47}$$

which describe indeed the scale factors of absorption of the light at the redshift conditions before (see Eq. (6.44)) and after (see Eq. (6.45)) the inflation, respectively.

Taking according to Eqs. (3.77) to (3.80) for the massive universe ($R \geq R_{\mathrm{Pl}}$) the constant vacuum mass density

$$\rho_{\mathrm{vac}} = \Omega_\Lambda \rho_{0C} = \Omega_\Lambda \frac{3H_0^2}{8\pi G_N} = 3.27 \cdot 10^3 (\mathrm{eV}/c^2)\,\mathrm{cm}^{-3} \tag{6.48}$$

as well as the constants (6.46) and (6.47) now as real integration constants, for the pressure $P = -\rho_{\mathrm{vac}} c^2$ (see Eq. (6.48)) and the cosmological "constant"

$\Lambda = 0$, the Friedmann-Lemaitre Equations (see Eqs. (2.4) and (2.5)) provide the present, exponential expansions (see also Eq. (2.23)):

$$\widetilde{R} = \widetilde{R}_0 \, \mathrm{e}^{\Omega_\Lambda^{1/2} H_0 (t-t_0)} = \widetilde{R}_0 \, \mathrm{e}^{(8\pi G_N \rho_{\mathrm{vac}}/3)^{1/2}(t-t_0)} = \widetilde{R}_0 \, \mathrm{e}^{(c^2\Lambda/3)^{1/2}(t-t_0)} \tag{6.49}$$

and

$$R = R_0 \, \mathrm{e}^{\Omega_\Lambda^{1/2} H_0 (t-t_0)} = R_0 \, \mathrm{e}^{(8\pi G_N \rho_{\mathrm{vac}}/3)^{1/2}(t-t_0)} = R_0 \, \mathrm{e}^{(c^2\Lambda/3)^{1/2}(t-t_0)} . \tag{6.50}$$

Here, in contrast to Ref. [1], Eq. (6.49) is correctly written.

By Eq. (3.79), for the massive universe, we have the cosmological "constant"

$$\Lambda = \frac{3\Omega_\Lambda}{R_0^2} = 1.087 \cdot 10^{-52} \, \mathrm{m}^{-2} . \tag{6.51}$$

Then, by generalization, via the transformation $T_{\mathrm{Pl}} \to T$ in Eq. (6.12), for the massive universe $(R \geq R_{\mathrm{Pl}})$, we find also its vacuum energy density as quantum gravity to

$$\rho_{\mathrm{vac}}(T) c^2 = \frac{1}{3} \frac{\pi^2}{15} \frac{(kT)^4}{(\hbar c)^3} \frac{1}{N(T)} . \tag{6.52}$$

Consequently, because of Eq. (6.18), for the massive universe, the cosmological "constant" can be assumed to

$$\Lambda = \widetilde{\Lambda} = \frac{8}{45} \pi^3 \frac{(kT)^4}{E_{\mathrm{Pl}}^2 (\hbar c)^2} \frac{1}{N(T)} . \tag{6.53}$$

Now, for the massive universe, we compare the radiation or particle energy density with the vacuum energy density (6.52). For this goal, we apply the data of the X and Y gauge bosons [$t = t_{\mathrm{X,Y}} = 2.67 \cdot 10^{-40}$ s (see Eq. (3.35)) and $T = T_{\mathrm{X,Y}} = 3.104 \cdot 10^{29}$ K (see Eq. (3.36)) as well as $N(T) = 160.75$ (see Table VI)]. For the solution (3.52), the particle energy density (3.50) is identical with the radiation energy density (see Eqs. (2.17) and (2.18)), so that we have

$$\rho_{\mathrm{r}} c^2 = \frac{1}{4} \rho_{0\mathrm{C}} c^2 \frac{R_0^2}{(ct)^2} = 3.519 \cdot 10^{117} \, \mathrm{eV \, cm^{-3}} . \tag{6.54}$$

In contrast to Eq. (6.54), the vacuum energy density (6.52) yields

$$\rho_{\text{vac}}(T)c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} = 9.09\cdot 10^{112}\,\text{eV}\,\text{cm}^{-3}\,, \tag{6.55}$$

i.e. it can be neglected in comparison with the energy density (6.54) at the normal description of the massive universe. However, it has influence on the massive universe. For this goal, using Eqs. (6.48) and (6.51), we can introduce the two conditions

$$\rho_{\text{vac}}(T)c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \geq \rho_{\text{vac}}c^2 = \Omega_\Lambda \rho_{0\text{C}}c^2 \tag{6.56}$$

and

$$\tilde{\Lambda} = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\text{Pl}}^2(\hbar c)^2}\frac{1}{N(T)} \geq \Lambda = \frac{3\Omega_\Lambda}{R_0^2}\,. \tag{6.57}$$

Using $N(T) = 3.362644$ (see, e.g., Refs [5, 7]), by the two conditions (6.56) and (6.57), we get the lower limiting temperature

$$T = 51.41\,\text{K}\,. \tag{6.58}$$

This temperature agrees excellently with the value $T = (1+z_\Lambda)T_0 = 51.45\,\text{K}$, where for $1+z_\Lambda = 18.90$ the influence of the vacuum energy density (6.48) begins (see Eq. (7.14)). Thus, we assume that the vacuum energy density (6.55) influences the present, exponential expansion, which therefore is considered.

Thus, we treat now this present, accelerated expansion. According to the dark mass density (6.48), the present, accelerated (acc) expansion begins at

$$1+z_{\text{acc}} = \frac{R_0}{R} = (2\Omega_\Lambda/\Omega_{\text{m}})^{1/3} = 1.632 \tag{6.59}$$

because of

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}\left(\rho + \frac{3P}{c^2}\right) = 0 \tag{6.60}$$

with $\rho = \rho_\Lambda + \rho_{\text{m}}$ from Eq. (2.21) and $P = -\rho_{\text{vac}}c^2 = -\Omega_\Lambda \rho_{0\text{C}}c^2$ [see Eqs. (3.77) to (3.80)].

According to Eq. (7.11), the present, accelerated expansion begun after the big bang at the time

$$t(z_{\text{acc}}) = 2.43\cdot 10^{17}\,\text{s} = 7.70\,\text{Gyr}\,. \tag{6.61}$$

By Eq. (7.12), this time yields the corresponding scale factor (see Eq. (6.45))

$$R = R(z_{\mathrm{acc}}) = R_0/(1+z_{\mathrm{acc}}) = 8.43\cdot 10^{25}\,\mathrm{m}\,. \tag{6.62}$$

According to Eqs. (6.50) and (6.62), the beginning of the present, accelerated expansion takes place before the time

$$t_{\mathrm{acc}} = t_0 - t = 2.72\cdot 10^{17}\,\mathrm{s} = 8.62\ \mathrm{Gyr}\,, \tag{6.63}$$

seen backward from the present age $t_0 = 4.358\cdot 10^{17}\,\mathrm{s}$ of the universe (see Table I). In other words, the time (6.63) is necessary to reach the present scale factor $R_0 = 1.375\cdot 10^{26}\,\mathrm{m}$ (see Eqs. (3.41) and (6.47) [z=0]) from the scale factor (6.62).

Between Eqs. (6.61) and (6.63), the discrepancy can be solved by Eq. (6.67) via a corresponding cosmological "constant" $\tilde{\Lambda}$ (see, e.g., Eq. (6.57)). For this goal, we apply, for example, Eq. (6.50), where the transformations $\Omega_\Lambda^{1/2} H_0 \to H$, $\rho_{\mathrm{vac}} \to \rho_{\mathrm{vac}}(T)$ or $\Lambda \to \tilde{\Lambda}$ are possible because the temperature has a constant value by the conditions (6.57) and (6.59) as well as (6.66).

Then, at conservation of the constant result (6.59), for example, in Eq. (6.50), we get the new condition

$$t_0 - \tilde{t} = \frac{\ln(1+z_{\mathrm{acc}})}{(c^2\tilde{\Lambda}/3)^{1/2}} = \frac{\ln(1+z_{\mathrm{acc}})}{(8/135\,\pi^3)^{1/2}(k^4T^4/N(T)E_{\mathrm{Pl}}^2\hbar^2c^4)^{1/2}} \le$$

$$\le t_0 - t = \frac{\ln(1+z_{\mathrm{acc}})}{\Omega_\Lambda^{1/2} H_0}\,, \tag{6.64}$$

so that we find the connection

$$(t_0 - \tilde{t})\,(c^2\tilde{\Lambda}/3)^{1/2} = (t_0 - \tilde{t})(8/135\,\pi^3)^{1/2}(k^4T^4/N(T)E_{\mathrm{Pl}}^2\hbar^2c^4)^{1/2} =$$

$$= (t_0 - t)\,\Omega_\Lambda^{1/2} H_0\,, \tag{6.65}$$

i.e. the duration $t_0 - \tilde{t}$ is given by

$$(t_0 - \tilde{t}) = (t_0 - t)\,\frac{\Omega_\Lambda^{1/2} H_0}{(c^2\tilde{\Lambda}/3)^{1/2}} =$$

$$= (t_0 - t)\,\frac{\Omega_\Lambda^{1/2} H_0}{(\frac{8}{135}\pi^3)^{1/2}(k^4 T^4/N(T)\,E_{\mathrm{Pl}}^2\,\hbar^2 c^4)^{1/2}}\,. \tag{6.66}$$

Consequently, because of the condition (6.57), the enlarged, present, accelerated expansion

$$R(z_{\mathrm{acc}}) = R_0\,\mathrm{e}^{(c^2\tilde{\Lambda}/3)^{1/2}(\tilde{t}-t_0)} \tag{6.67}$$

is defined by a smaller duration $t_0 - \tilde{t}$ than $t_0 - t$ (see Eqs. (6.50) and (6.67)) at conservation of the redshift condition (6.62), i.e. the influence of the general vacuum energy density $\rho_{\mathrm{vac}}(T)\,c^2$ or the cosmological "constant" $\tilde{\Lambda}$ on the massive universe is not negligible.

Therefore, we can solve the discrepancy between Eqs. (6.61) and (6.63) if in Eq. (6.66) we select $\tilde{t} = t(z_{\mathrm{acc}}) = 2.43\cdot 10^{17}\,\mathrm{s}$, so that for $t_0 - t = 2.72\cdot 10^{17}\,\mathrm{s}$ Eq. (6.66) provides $(c^2\tilde{\Lambda}/3)^{1/2} = 2.55\cdot 10^{-18}\,\mathrm{s}^{-1}$ with $\tilde{\Lambda} = 2.16\cdot 10^{-52}\,\mathrm{m}^{-2}$ or $\rho_{\mathrm{vac}}(T)\,c^2 = 6.50\cdot 10^3\,\mathrm{eV\,cm}^{-3}$ for $N(T) = 3.362644$ at $T = 61\,\mathrm{K}$.

Thus, by this constant temperature $T = 61\,\mathrm{K}$, the influence of the dark energy begins a little earlier at the present, accelerated expansion. By this fact, we have explained this discrepancy for Eqs. (6.61) and (6.63).

Using the data of Tables III to V, all results, derived in this Sec. 6.4 for the massive universe, are also valid for the massive anti-universe.

6.5 Several conclusions from the vacuum energy density or the cosmological "constant"

The four-dimensional quantum gravity, which is explicitly defined completely by Eqs. (6.34) and (6.37) as well as (6.39) to (6.43), gives uniquely the vacuum energy density (6.37) or the cosmological "constant" (6.39) in the space range

$R_{BB} \leq R \leq R_{Pl}$ of the massless universe, i.e. we have found a simple quantum-theoretical, four-dimensional description for the gravitation.

The expressions (6.41) and (6.43) mean clearly that the massless universe is exclusively characterized by the general vacuum energy density or the radiation (photon (particle)) energy density, respectively. They are identically described by Eq. (6.37).

Generally, the vacuum energy density, which is represented by the cosmological "constant", decreases by the expansion (see Secs. 3.2.5 as well as 6.1 to 6.4), since at this the distance R increases. The vacuum energy density is again connected with the dark energy, which is repulsive, i.e. the dark energy determines the expansion dynamics of the massive universe, so that the dark energy is responsible for the accelerated expansion of the massive universe [see Eqs. (3.77) to (3.80) as well as (6.59) to (6.67)].

More generally, because by the expansion the vacuum energy density decreases, the dark energy must disappear in the future, i.e. the accelerated expansion of the massive universe must end by a negative acceleration (see Eqs. (7.26) and (7.27)). We have uniquely determined the general vacuum energy density or the cosmological "constant" for the massless ($R_{BB} \leq R \leq R_{Pl}$) [see Sec. 6.3] and the massive ($R \geq R_{Pl}$) [see Sec. 6.4] universe, i.e. we have found a simple solution for the four-dimensional quantum gravity. At $R = R_{Pl}$, the excellent agreement of the vacuum energy densities (see Sec. 3.2.5), derived by the new inflation model (see Sec. 3.2.4) and the new quantum-statistical photon energy density (see Sec. 6.1), confirms the correctness of the estimation of the rest energies of the light neutrinos (see Eqs. (3.8) to (3.11)) and the SUSY GUT particles [X and Y gauge bosons (see Eq. (3.34)) as well as magnetic monopoles (see Eq. (3.44))].

In the massive universe the radiation energy density and the particle energy density are identical. They dominate clearly the quantum vacuum in form of the vacuum energy density (compare Eqs. (6.54) and (6.55)), so that it can be neglected at the normal description of the massive universe except the present, accelerated expansion and the final state of the universe. At the present, accelerated expansion, its small influence is demonstrated in Sec. 6.4. In Sec. 8.4, we show the dominant influence of the vacuum energy density in the final state of the massive universe.

By the process described in Sec. 6.4, we have introduced a new thermal equilibrium between photons with the thermal energy kT as well as particles (P) with the rest energy $E_0(\mathrm{P})$ or antiparticles ($\overline{\mathrm{P}}$) with the rest energy $E_0(\overline{\mathrm{P}})$. At $R = \pm R_{\mathrm{Pl}}$, for $kT = E_{\mathrm{Pl}}$, all particles of the universe or antiparticles of the anti-universe have the Planck energy as relativistic energy, i.e. $kT >> E_0(\mathrm{P})$ or $kT >> E_0(\overline{\mathrm{P}})$. Thus, for $kT = E_0(\mathrm{P})$ or $kT = E_0(\overline{\mathrm{P}})$, the kinetic energy of the particles or antiparticles is zero. This temperature of the photons determines the redshift condition $1 + z = T/T_0$, which defines the expansion dynamics of the universe via its mass density (see, e.g. Eq. (2.6) as well as Eqs. (2.4) and (2.5)).

Therefore, for $R \geq R_{\mathrm{Pl}}$, in this particle mixture, only the events, which are determined by the reaction laws of nuclear and elementary physics, can take place, i.e. we have no complete mixture of non-separated particles and antiparticles in the universe, so that the small number of non-separated particles and antiparticles, produced by these reaction laws, must again disappear by annihilation (see also the paragraph after Eq. (3.36)). This new result, which solves simply the separation of matter and antimatter, does not agree with the old interpretation, in which for the universe the particles and antiparticles are produced coincidently by photons with the thermal energy $kT = E_0(\mathrm{P}) + E_0(\overline{\mathrm{P}})$, so that this old interpretation has a problem at the separation of matter and antimatter.

Using again the data of Tables III to V, all results, considered in this Sec. 6.5 for the total universe, are also valid for the total anti-universe.

7 New results for the massive universe and anti-universe

7.1 The final state of the massive universe as well as the end of the present, accelerated expansion by a mean, negative acceleration

The modern relativistic cosmology is based on the General Relativity ($R_{\mathrm{Pl}} \leq R < \infty$) and the quantum theory ($0 < R < \infty$). However, the different

ranges of validity for the General Relativity and the quantum theory lead to an incompatibility between these theories, so that the Friedmann-Lemaitre equations (see Eqs. (2.4) and (2.5)), which are based on the (classical) General Relativity, have a limit of their applicability on massive particles [1, 2, 4]. Then, for massive particles of the energy E, the uncertainty relation $R = \hbar c/2E$, multiplied by their Schwarzschild radius $R = 2G_N E/c^4$ (see also Eq. (6.26)), which describes these massive particles (matter), yields this limit [1, 2, 4] as the so-called Planck lenght

$$R_{\mathrm{Pl}} = c\,t_{\mathrm{Pl}} = \sqrt{\frac{\hbar G_N}{c^3}} = (1.616200 \pm 0.000096)\cdot 10^{-35}\ \mathrm{m}\,. \tag{7.1}$$

Then, derived Planck units are the Planck time, the Planck energy and the Planck temperature:

$$t_{\mathrm{Pl}} = \frac{R_{\mathrm{Pl}}}{c} = \sqrt{\frac{\hbar G_N}{c^5}} = (5.39106 \pm 0.00032)\cdot 10^{-44}\ \mathrm{s}\,, \tag{7.2}$$

$$E_{\mathrm{Pl}} = \frac{\hbar c}{R_{\mathrm{Pl}}} = \frac{\hbar}{t_{\mathrm{Pl}}} = \sqrt{\frac{\hbar c^5}{G_N}} = (1.220932 \pm 0.000073)\cdot 10^{28}\ \mathrm{eV}\,, \tag{7.3}$$

$$T_{\mathrm{Pl}} = \frac{E_{\mathrm{Pl}}}{k} = \sqrt{\frac{\hbar c^5}{G_N k^2}} = (1.416833 \pm 0.000086)\cdot 10^{32}\ \mathrm{K}\,. \tag{7.4}$$

The above-mentioned incompatibility between the General Relativity and the quantum theory can only be solved by the quantum gravity, which must reconcile the gravitation with the quantum theory (see Sec. 6.3).

For the estimation of the Planck energy (see Eq. (7.3)), we can also as usually [1, 2, 4, 10] consider two particles with the mass $m = E/c^2$, which have the distance of the reduced Compton wavelength $\lambda_{\mathrm{C}} = \hbar c/E$. Then, the gravitation yields the potential energy $V = G_N m^2/\lambda_{\mathrm{C}} = G_N E^3/c^5\hbar$. An upper limit for the correctness of this classical approach gives the assumption that this potential cannot exceed the limiting energy $E = E_{\mathrm{Pl}} = \sqrt{\hbar c^5/G_N}$ [Planck energy (see Eq. (7.3))] because according to the indeterminacy principle the quantum fluctuations of the distance cause an energy uncertainty in order of magnitude of this Planck energy, so that it is not possible to describe these

particles by a well-defined rest energy. Because of $E_0(\mathrm{M}) = 6.849 \cdot 10^{26}\,\mathrm{eV}$, the rest energy of the magnetic monopoles, which is well-defined because of $E_0(\mathrm{M}) << E_{\mathrm{Pl}}$, is again connected with the scale factor of the Planck length $R_{\mathrm{Pl}} = ct_{\mathrm{Pl}} = \sqrt{\hbar G_N / c^3} = \lambda_{\mathrm{C}}$ via their relativistic energy $E_{\mathrm{Pl}} = \hbar c / R_{\mathrm{Pl}} = \hbar / t_{\mathrm{Pl}}$ (see Eq. (3.72)) or their redshift condition (see Eq. (3.48)).

In contrast to the large mass of the monopoles ($E_0(\mathrm{M}) = 6.849 \cdot 10^{26}\,\mathrm{eV}$), which is connected with the Planck length via their redshift condition (see above), we can assume that the small mass of the electron neutrinos ($E_0(\nu_{\mathrm{e}}) = 1.589 \cdot 10^{-3}\,\mathrm{eV}$) defines an upper limiting value of the scale factor for the Friedmann-Lemaitre equations as the final state of the universe via their blueshift condition [1, 2, 4].

Thus, we assume that this final state can be estimated as follows. Because of $\Omega_{\mathrm{m}} = \{\Omega_{\mathrm{dm}} + \Omega_\nu(\nu_\tau) + \Omega_\nu(\nu_\mu) + \Omega_\nu(\nu_{\mathrm{e}})\} + \Omega_{\mathrm{b}}$ and $N(T) = 3.362644$, for this goal, we use the redshift condition of the matter-radiation equality (see Tables I, II and V):

$$1 + z_{\mathrm{eq}} = \frac{2\Omega_{\mathrm{m}}}{N(T)\,\Omega_\gamma} = \\ = \frac{2\left(\{\Omega_{\mathrm{dm}} + \Omega_\nu(\nu_\tau) + \Omega_\nu(\nu_\mu) + \Omega_\nu(\nu_{\mathrm{e}})\} + \Omega_{\mathrm{b}}\right)}{N(T)\,\Omega_\gamma}, \tag{7.5}$$

so that by Eq. (3.26) we can also assume

$$1 + z(\nu_{\mathrm{e}}) = \frac{2\{\Omega_{\mathrm{m}} - (\{\Omega_{\mathrm{dm}} + \Omega_\nu(\nu_\tau) + \Omega_\nu(\nu_\mu)\} + \Omega_{\mathrm{b}})\}}{N(T)\,\Omega_\gamma} = \\ = \frac{2\Omega_\nu(\nu_{\mathrm{e}})}{N(T)\,\Omega_\gamma} = 0.406^{+0.020}_{-0.025}\,. \tag{7.6}$$

Then, the neutrinos must be considered together with the photons (see Eq. (2.8)). However, the matter-dominated universe expands for the redshift condition $1 + z_{\mathrm{eq}} >> 1 + z \geq 1$, i.e., the scale factor R increases up to R_0, i.e. $R \leq R_0$. At $1 + z \leq 1$, we have a blueshift, so that R increases from R_0, i.e. $R \geq R_0$.

As above-mentioned, we can assume that the blueshift $1+z(\nu_e)=0.406$ determines a final state of the universe, where the particles do not move more practically. Therefore, a mean, negative acceleration, which effectively must finish with the present, exponential expansion, must exist. After the end of this present, accelerated expansion, we observe a stop of the particles which have moved still further as a result of a slow, linear expansion by the astronomical unit changing up to the final state of the universe. This astronomical unit changing is calculated in Sec 7.2.

Now, we determine the values of the final state and the mean, negative acceleration as well as the end of the present, accelerated expansion as follows.

At Eq. (2.20), for $z_{eq} >> z >> 1$, we can neglect Ω_Λ, so that the matter-dominated universe is described by the Friedmann equation of the form

$$\left(\frac{\dot{R}}{R}\right)^2 = \Omega_m H_0^2 \left(\frac{R_0}{R}\right)^3 . \tag{7.7}$$

Eq. (7.7) has the solutions

$$R = (3/2)^{2/3}\, \Omega_m^{1/3} R_0 (H_0 t)^{2/3} \tag{7.8}$$

and

$$t = t(z) = \frac{2}{3 H_0 \left[\Omega_m (1+z)^3\right]^{1/2}} . \tag{7.9}$$

However, for Eq. (2.21), at $z << z_{eq}$, we cannot more neglect Ω_Λ, so that by Eq. (2.5) the matter-dominated universe is defined by

$$\left(\frac{\dot{R}}{R}\right)^2 = H_0^2 (\Omega_m \left(\frac{R_0}{R}\right)^3 + \Omega_\Lambda) \tag{7.10}$$

with the solutions

$$t = t(z) = \frac{2}{3 H_0 \Omega_\Lambda^{1/2}} \ln \frac{\sqrt{\Omega_\Lambda (1+z)^{-3}} + \sqrt{\Omega_\Lambda (1+z)^{-3} + \Omega_m}}{\sqrt{\Omega_m}} \tag{7.11}$$

and

$$\frac{R}{R_0} = \frac{1}{1+z} = \left(\frac{\Omega_{\mathrm{m}}}{\Omega_\Lambda}\right)^{1/3} \left[\sinh(3/2\,\Omega_\Lambda^{1/2} H_0 t)\right]^{2/3} =$$

$$= \left(\frac{\Omega_{\mathrm{m}}}{\Omega_\Lambda}\right)^{1/3} \left[\frac{\mathrm{e}^{3/2\,\Omega_\Lambda^{1/2} H_0 t} - \mathrm{e}^{-3/2\,\Omega_\Lambda^{1/2} H_0 t}}{2}\right]^{2/3} . \tag{7.12}$$

By Eqs. (7.9) and (7.11), we can form the equation

$$\left(\frac{\Omega_\Lambda}{\Omega_{\mathrm{m}}}\right)^{1/2} \frac{1}{(1+z_\Lambda)^{3/2}} =$$

$$= \ln \frac{\sqrt{\Omega_\Lambda (1+z_\Lambda)^{-3}} + \sqrt{\Omega_\Lambda (1+z_\Lambda)^{-3} + \Omega_{\mathrm{m}}}}{\sqrt{\Omega_{\mathrm{m}}}} . \tag{7.13}$$

Using the corresponding density parameters in Table I, Eq. (7.13) yields the redshift condition

$$1 + z_\Lambda = 18.90^{+0.48}_{-0.49} , \tag{7.14}$$

so that in Eqs. (2.20) or (2.21) the influence of the dark energy (Ω_Λ) begins for $z_\Lambda = 18.90$ according to Eqs. (7.9) or (7.11) at

$$t = t(z_\Lambda) = 6.63 \cdot 10^{15}\ \mathrm{s} = 0.210\ \mathrm{Gyr} . \tag{7.15}$$

Then, taking Eqs. (3.59) as well as (7.6), according to Refs. [1, 2, 4], for the scale factor of the final state of the universe (see above), we get

$$R = R(\nu_{\mathrm{e}}) = \frac{R_0}{1+z(\nu_{\mathrm{e}})} = (3.39^{+0.27}_{-0.23}) \cdot 10^{26}\ \mathrm{m} . \tag{7.16}$$

By Eq. (7.14), for the beginning influence of the dark energy, we find its minimum scale factor [1, 2, 4]:

$$R = R_\Lambda = \frac{R_0}{1+z_\Lambda} = (7.28 \pm 0.32) \cdot 10^{24}\ \mathrm{m} . \tag{7.17}$$

Because of the expressions (7.16) and (7.17), which define the range of influence of the dark energy (Ω_Λ), for the mean, negative acceleration (MNA), we must assume the average of their redshift conditions [1, 2, 4]:

$$1+z_{\mathrm{MNA}}=\frac{R_0}{\sqrt{R_\Lambda R(\nu_{\mathrm{e}})}}=\left([1+z_\Lambda][1+z(\nu_{\mathrm{e}})]\right)^{1/2}=2.77^{+0.10}_{-0.12}, \tag{7.18}$$

so that we obtain the corresponding scale factor [1, 2, 4]:

$$R=R_{\mathrm{MNA}}=\frac{R_0}{1+z_{\mathrm{MNA}}}=(4.96^{+0.31}_{-0.27})\cdot 10^{25}\ \mathrm{m}. \tag{7.19}$$

Consequently, by Eqs. (2.5) and (2.21), via Eqs. (7.18) and (7.19), for the matter-dominated universe, [where because of $1+z_{\mathrm{MNA}}=2.77$ we can still assume $P=0$, i.e. $\Omega_\Lambda << \Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^3$ in Eq. (2.21)], the mean negative acceleration [1, 2, 4] is effectively defined by

$$\ddot{R}_{\mathrm{MNA}}=a=-\frac{1}{2}H_0^2\left[\Omega_{\mathrm{m}}(R_0/R_{\mathrm{MNA}})^3+\Omega_\Lambda\right]R_{\mathrm{MNA}}=$$
$$=(-8.71^{+0.76}_{-0.87})\cdot 10^{-8}\ \mathrm{cm\ s^{-2}}. \tag{7.20}$$

According to Eq. (7.11), this negative acceleration (7.20) exists from the time

$$t=t_{\mathrm{MNA}}=t(z_{\mathrm{MNA}})=1.162\cdot 10^{17}\ \mathrm{s}=3.68\ \mathrm{Gyr}. \tag{7.21}$$

The upper assumption $P=0$ is reasonable because for $P\neq 0$ according to Eqs. (6.59) and (6.62), the present, accelerated (acc) expansion begins at

$$1+z_{\mathrm{acc}}=\frac{R_0}{R}=1.632 \tag{7.22}$$

and

$$R=R_0/(1+z_{\mathrm{acc}})=8.43\cdot 10^{25}\mathrm{m}, \tag{7.23}$$

i.e. according to Eq. (7.11) the present, accelerated expansion begun at the time、

$$t=t(z_{\mathrm{acc}})=2.43\cdot 10^{17}\ \mathrm{s}=7.70\ \mathrm{Gyr} \tag{7.24}$$

after the big bang.

The value (7.20) agrees excellently with the observed [26] Pioneer (P) anomaly

$$\ddot{R}_{\mathrm{P}}=a_{\mathrm{P}}=(-8.74\pm 1.33)\cdot 10^{-8}\ \mathrm{cm\ s^{-2}}. \tag{7.25}$$

This excellent agreement is an evidence for the correctness of $E_0(\nu_{\mathrm{e}})$ and $\Omega_\nu(\nu_{\mathrm{e}})$ as well as of the assumption of a final state of the universe. In Sec 7.3, we will return to this final state.

Consequently, Eq. (7.20) confirms the significance of the electron neutrino for the universe.

With that, we have solved the mystery of the Pioneer anomaly [1-3, 26, 27]. Therefore, the thermal recoil pressure computer simulation [28] is doubtful because it explains the Pioneer anomaly as an unreal physical effect.

By Eq. (2.5), for $\Lambda = 0$, according to $\rho = \rho_{\text{vac}} c^2 = \Omega_\Lambda \rho_{0\text{C}} c^2$ (see Eq. (6.48)) and $P = -\rho_{\text{vac}} c^2 = -\Omega_\Lambda \rho_{0\text{C}} c^2$, the acceleration of the present, accelerated expansion is determined by

$$\ddot{R} = \Omega_\Lambda H_0^2 R . \tag{7.26}$$

Consequently, for the end of this present, accelerated expansion, the effective (eff) equilibrium condition $\ddot{R} + \ddot{R}_{\text{MNA}} = 0$ yields the effective scale factor

$$R_{\text{eff}} = \frac{-\ddot{R}_{\text{MNA}}}{\Omega_\Lambda H_0^2} = 2.67 \cdot 10^{26}\ \text{m} . \tag{7.27}$$

We will return to this problem (7.27) in Sec. 7.3.

Using the data of Tables III to V, all results, derived in this Sec. 7.1, are also valid for the anti-universe.

Therefore, since the universe and the anti-universe are completely symmetric, the measured flyby anomaly, which in the works [2-4] is attributed to the mean negative acceleration, must be observed also by the antihuman on the anti-earth in the anti-universe because of a mean positive acceleration. This flyby is a strong argument for the existence of the Pioneer anomaly as a real physical effect (see above).

7.2 The astronomical unit changing

An astronomical unit changing was observed by two independent researcher groups, analysing radio echoes from the planets [29, 30]. According to Krasinsky and Brumberg [29], the astronomical unit changing was found to

$$\frac{d}{dt}\text{AU} = (15 \pm 4)\ \text{cm yr}^{-1} , \tag{7.28}$$

where $\mathrm{AU} = 1.49597870700(3)\cdot 10^{11}\,\mathrm{m}$ is the astronomical unit [9].

Because of Eqs. (2.21) and (7.19), we have

$$\rho = \rho_{\mathrm{MNA}} = \frac{3H_0^2}{8\pi G_N}\left[\Omega_{\mathrm{m}}(R_0/R_{\mathrm{MNA}})^3 + \Omega_\Lambda\right], \tag{7.29}$$

so that because of $P = 0$ (see Eq. (6.60)) as well as $\mathrm{k} = 0$ (see Eq. (2.5)) and $\Lambda = 0$ the Friedmann-Lemaitre Equations (see Eqs. (2.4) and (2.5)) yield

$$\frac{1}{2}\left(\frac{\dot{R}_{\mathrm{MNA}}}{R_{\mathrm{MNA}}}\right)^2 + \frac{\ddot{R}_{\mathrm{MNA}}}{R_{\mathrm{MNA}}} = 0\,. \tag{7.30}$$

Therefore, we use the equation

$$\frac{1}{2}\left(\frac{\dot{R}_{\mathrm{MNA}}}{R_{\mathrm{MNA}}}\right)^2 = \frac{4\pi G_N}{3}\rho_{\mathrm{MNA}} \tag{7.31}$$

for the introduction of the velocity

$$v = \frac{1}{2}\frac{\dot{R}_{\mathrm{MNA}}^2}{H_0\,R_{\mathrm{MNA}}} = \frac{1}{2}H_0\left[\Omega_{\mathrm{m}}(R_0/R_{\mathrm{MNA}})^3 + \Omega_\Lambda\right]R_{\mathrm{MNA}}\,. \tag{7.32}$$

Thus, via Eq. (7.32), the mean astronomical unit changing [1-4] can be defined by

$$\frac{d}{dt}\mathrm{AU} = \frac{1}{2}\frac{\dot{R}_{\mathrm{MNA}}^2}{H_0 R_{\mathrm{MNA}}}\mathrm{corr(AU)} =$$

$$= \frac{1}{2}H_0\left(\Omega_{\mathrm{m}}\left(\frac{R_0}{R_{\mathrm{MNA}}}\right)^3 + \Omega_\Lambda\right)R_{\mathrm{MNA}}\,\mathrm{corr(AU)}\,, \tag{7.33}$$

where the correction constant corr(AU) is necessary as a proportionality factor, since the radio echoes, which are observed on the surface of the earth, are reflected on the surface of the planets. Consequently, the radio signals run twice the distance $\Delta\mathrm{AU} = \mathrm{AU} - (R_{\mathrm{earth}} + R_{\mathrm{sun}})$, so that this corrective constant must be assumed to

$$\mathrm{corr(AU)} = 2\frac{\mathrm{AU} - \Delta\mathrm{AU}}{R_0} =$$

$$= 2\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} = 2\times(5.105 \pm 0.093)\cdot 10^{-18}\,, \qquad (7.34)$$

where $R_{\mathrm{earth}} = 6.378137\cdot 10^6\,\mathrm{m}$ and $R_{\mathrm{sun}} = (6.9551 \pm 0.0004)\cdot 10^8\,\mathrm{m}$ are the equatorial radii of the earth and the sun [9]. Plausibly, the denominator of this corr(AU) must be defined by the scale factor R_0 for the present-day universe ($z = 0$). Thus, by Eqs. (7.19) as well as (7.33) and (7.34), we find the mean astronomical unit changing [1-4] to

$$\frac{d}{dt}\mathrm{AU} = c\left[\Omega_{\mathrm{m}}(1+z_{\mathrm{MNA}})^2 + \Omega_\Lambda/(1+z_{\mathrm{MNA}})\right]\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} =$$

$$= (2.521^{+0.18}_{-0.21})\cdot 10^{18}\,\mathrm{cm\,yr^{-1}}\,\frac{R_{\mathrm{earth}} + R_{\mathrm{sun}}}{R_0} = (12.9^{+1.2}_{-1.3})\,\mathrm{cm\,yr^{-1}}\,. \qquad (7.35)$$

The theoretical value (7.35) agrees excellently with the result (7.28) of the analysis of all available radiometric measurements in the time span 1961-2003 [29].

This result is in accordance with a later analysis of the solar-system anomalies in Ref. [31], which for the astronomical unit changing permits the limiting conditions $0 \le {}^{d}\!/_{dt}\,\mathrm{AU} \le 20\,\mathrm{cm\,yr^{-1}}$. The very small velocity (7.35) is presently very difficult to observe because it is covered up by the present, accelerated expansion.

Using the data of Tables III to V, all results, derived in this Sec. 7.2 for the massive universe, are also valid for the massive anti-universe.

7.3 The future of the massive universe by a slow linear expansion

Using Eqs. (2.23) and (7.27), the effective equilibrium takes place at the time

$$t_{\mathrm{eff}} = t_0 + \frac{1}{\Omega_\Lambda^{1/2} H_0}\ln\frac{R_{\mathrm{eff}}}{R_0} = 8.034\cdot 10^{17}\,\mathrm{s} = 25.46\,\mathrm{Gyr}\,. \qquad (7.36)$$

Now, for Eq. (7.27), we estimate the influence of the astronomical unit changing (auc) of Eq. (7.35) on the exponential (exp) scale factor $R_{\exp}$, i.e. we must consider the relation

$$R_{\text{eff}} = R_{\exp} + \Delta R_{\text{auc}} = 2.67 \cdot 10^{26}\,\text{m}\,. \tag{7.37}$$

For this goal, we assume as relative changing

$$\frac{\Delta R_{\text{auc}}}{R_{\text{MNA}}} = \frac{R_{\text{eff}} - R_{\text{MNA}}}{R_{\text{MNA}}} = \frac{\Delta t_{\text{auc}}}{\text{AU}} \frac{d}{dt}\text{AU} = \frac{t_{\text{eff}} - t_{\text{MNA}}}{\text{AU}} \frac{d}{dt}\text{AU} \tag{7.38}$$

with

$$\frac{1}{\text{AU}} \frac{d}{dt}\text{AU} = 2.733 \cdot 10^{-20}\,\text{s}^{-1}. \tag{7.39}$$

Then, using Eqs. (7.19) and (7.21) as well as (7.36) and (7.39), we get

$$\Delta R_{\text{auc}} = R_{\text{MNA}} \frac{t_{\text{eff}} - t_{\text{MNA}}}{\text{AU}} \frac{d}{dt}\text{AU} = 9.32 \cdot 10^{23}\,\text{m}\,. \tag{7.40}$$

In first approximation, the result (7.40) can be neglected in the relation (7.37).

At $R_{\text{eff}} = 2.67 \cdot 10^{26}\,\text{m}$ (see Eqs. (7.27) and (7.37)), we observe the end of the present, accelerated expansion, i.e. in future, the present exponential expansion, which is determined by the dark energy, gives way to a slow linear expansion of the universe by the astronomical unit changing. Therefore, we can estimate the age of the universe for its final (f) state because we know its scale factor $R_{\text{f}} = R(\nu_{\text{e}}) = 3.39 \cdot 10^{26}\,\text{m}$ (see Eq. (7.16)), i.e. similar to Eq. (7.38) we have here the total relative changing

$$\frac{\Delta R_{\text{auc}}}{R_{\text{MNA}}} = \frac{R_{\text{f}} - R_{\text{MNA}}}{R_{\text{MNA}}} = \frac{\Delta t_{\text{auc}}}{\text{AU}} \frac{d}{dt}\text{AU} = \frac{t_{\text{f}} - t_{\text{MNA}}}{\text{AU}} \frac{d}{dt}\text{AU}\,. \tag{7.41}$$

Consequently, via the results (7.19) and (7.21), for the age of the final state of the universe, Eq. (7.41) yields

$$t_{\text{f}} = t_{\text{MNA}} + \frac{R_{\text{f}} - R_{\text{MNA}}}{R_{\text{MNA}}} \left(\frac{1}{\text{AU}} \frac{d}{dt}\text{AU} \right)^{-1} =$$

$$= 2.136 \cdot 10^{20}\,\text{s} = 6768\,\text{Gyr}\,. \tag{7.42}$$

This age $6768\,\text{Gyr}$, determined by the slow linear expansion of the universe [1], is much greater than the age $t = t_{\exp}(\nu_{\text{e}}) = 29.63\,\text{Gyr}$ [2-4], which is alone

defined by the present, exponential expansion of the universe, i.e. for $R = R(\nu_e) = 3.39 \cdot 10^{26}\,\mathrm{m}$ (see Eq. (7.16)) in Eq. (2.23).

However, to the result (7.42), where we must still take into account the influence of the general vacuum energy density or the corresponding cosmological "constant" of the massive universe, we will return in Sec. 8.2.

For the future, in a time span $\Delta\tau \approx 200\,\mathrm{yr}$, the changing of the astronomical unit (see Eq. (7.35)) is well measurable [1-4] because we have at this directly an AU changing of about $26\,\mathrm{m}$ and the error of the present AU determination is only $3\,\mathrm{m}$ (see AU after Eq. (7.28)). If in the future this conclusion can be confirmed, the Pioneer anomaly and the astronomical unit changing are uniquely proved, so that they can be counted to the most important astronomical discoveries within living memory.

Using the data of Tables III to V, all results, derived in this Sec. 7.3 for the massive universe, are also valid for the massive anti-universe.

7.4 The Redshift values of the reionization

Much information about the intergalactic medium (IGM), which permeates the space between the galaxies, has been obtained through its absorption of light from distant quasars, whereby one of the most important tests of the presence of a diffuse component of the intergalactic neutral (mainly hydrogen (HI)) gas is the Gunn-Peterson effect [1-5].

This effect uses the fact that in the intergalactic neutral gas the density of these HI atoms, defined by $n_{\mathrm{HI}}(z) \approx 2.42 \cdot 10^{-11} \tau(z)\, h\, E(z)\ \mathrm{cm}^{-3}$ {in Ref. [5] for $E(z) = \left[\Omega_\Lambda + \Omega_m (1+z)^3 + \Omega_r (1+z)^4\right]^{1/2}$ (see Ref. [1]}, can be estimated according to Ref. [5] by the measurement of its optical depth $\tau(z)$ from flux decrement in quasar spectra at the wavelength $\lambda_{\mathrm{Ly}\alpha} = 121.6\,\mathrm{nm}$ in $\mathrm{Ly}\alpha$ absorption (of the neutral hydrogen), which has a very large cross-section for

$$T_{\mathrm{IGM}} << T_{\mathrm{Ly}\alpha} = \frac{2\pi\hbar c}{k\,\lambda_{\mathrm{Ly}\alpha}} = 1.183\cdot 10^5\,\mathrm{K}\,. \tag{7.43}$$

The observations show that at a redshift of order $z \approx 5\text{-}6$ the neutral hydrogen, left over from the time of recombination, becomes reionized by this ultraviolet light ($\lambda_{\mathrm{Ly}\alpha} = 121.6\,\mathrm{nm}$) from the massive stars (quasars) [5, 7].

For example, the condition (7.43) can be fulfilled by the neutrinos [1-4] if they determine T_{IGM}.

Thus, via Eqs. (3.27) to (3.29), analogously to Eqs. (7.5) and (7.6), we assume that shortly prior to and after the value $z_{\mathrm{MNA}} = 1.77$ (see Eq. (7.18)) the redshifts z_{reion} of the neutrinos

$$z_{\mathrm{reion}}(\textstyle\sum_i \nu_i) = \frac{2\sum_i \Omega_\nu(\nu_i)}{N(T)\,\Omega_\gamma} - 1 = 14.27^{+0.37}_{-0.33}\,, \tag{7.44}$$

$$z_{\mathrm{reion}}(\nu_\tau) = \frac{2\,\Omega_\nu(\nu_\tau)}{N(T)\,\Omega_\gamma} - 1 = 11.60^{+0.31}_{-0.26} \tag{7.45}$$

and

$$z_{\mathrm{reion}}(\nu_\mu) = \frac{2\,\Omega_\nu(\nu_\mu)}{N(T)\,\Omega_\gamma} - 1 = 1.263^{+0.036}_{-0.042} \tag{7.46}$$

determine the beginning, the half and the end of the reionization [1-4], respectively.

This assumption is supported by the neutrino temperatures

$$T(\textstyle\sum_i \nu_i) = \left\{1 + z_{\mathrm{reion}}(\sum_i \nu_i)\right\} T_0 = 41.62\,\mathrm{K}\,, \tag{7.47}$$

$$T(\nu_\tau) = \left\{1 + z_{\mathrm{reion}}(\nu_\tau)\right\} T_0 = 34.34\,\mathrm{K} \tag{7.48}$$

and

$$T(\nu_\mu) = \left\{1 + z_{\mathrm{reion}}(\nu_\mu)\right\} T_0 = 6.168\,\mathrm{K}\,, \tag{7.49}$$

since they fulfil the condition (7.43). Then, the neutrinos are indirectly connected with the photons via $N(T) = 3.362644$ (see Eq. (2.8)). Then, the mean redshift condition

$$\overline{1 + \bar{z}_{\mathrm{reion}}} = \\ = \left(\left[1 + z_{\mathrm{reion}}(\textstyle\sum_i \nu_i)\right] \times \left[1 + z_{\mathrm{reion}}(\nu_\tau)\right] \times \left[1 + z_{\mathrm{reion}}(\nu_\mu)\right]\right)^{1/3} = 7.58\,, \tag{7.50}$$

which yields $\bar{z} = 6.58$, must agree approximately with the above-mentioned redshift $z \approx 5\text{-}6$.

Therefore, the assumptions (7.44) to (7.46) show the significance of these neutrinos for the universe. The value (7.45) agrees excellently with the redshifts of the half reionization in the Tables I and II, which were found on completely other way. This fact is a strong argument for the correctness of the assumptions (7.44) and (7.46) as the beginning and the end of the reionization, respectively.

Using the data of Tables III to V, all results, derived in this Sec. 7.4 for the massive universe, are also valid for the massive anti-universe.

8 The transition from the final state of the massive universe (anti-universe) to the big bang

8.1 Parameters of the final state of the massive universe (anti-universe) and the big bang

According to Ref. [1], in this chapter, we calculate the parameters of the big bang in the total (massless and massive) universe.

At the time $t_{\text{eff}} = 8.034 \cdot 10^{17}\,\text{s} = 25.46\,\text{Gyr}$ (see Eq. (7.36)) or the scale factor $R_{\text{eff}} = 2.67 \cdot 10^{26}\,\text{m}$ (see Eq. (7.27)), we have the end of the present, accelerated expansion, so that here the final value of the dark (d) energy E_{d} is found via the vacuum energy density $\rho_{\text{vac}}\, c^2 = \Omega_\Lambda \rho_{0\text{C}}\, c^2 = 3.27 \cdot 10^3\,\text{eV cm}^{-3}$ (see Eq. 6.48)) to

$$E_{\text{d}} = \Omega_\Lambda \rho_{0\text{C}} c^2 \tfrac{4}{3} \pi d_{\text{eff}}^3 , \tag{8.1}$$

where the distance d_{eff} is defined by

$$d_{\text{eff}} = R_{\text{eff}}\, r_{\text{eff}} = R_{\text{eff}}\, r \tag{8.2}$$

with $r = r_{\text{eff}}$ as the dimensionless, time-independent, comoving coordinate distance (see Refs. [1, 5, 7, 10]).

Using the hypothesis of the joint origin of the dark matter and dark energy by the three sterile neutrino types $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ (see Sec. 5), we can assume that this dark energy E_{d} results from the decay of these three sterile neutrinos types.

Consequently, according to Sec. 5, the half, dark energy $\frac{1}{2}E_{\mathrm{d}}$ must be defined by massless photons, which arise from the decay of these sterile dark energy neutrinos ($\hat{\nu}$) by the gravitation in photons (γ) with the total, greatest possible, thermal energy

$$kT = \tfrac{1}{2}E_{\mathrm{d}} = \sum_{\gamma}\left\langle kT_{\gamma}\right\rangle = \sum_{\gamma}\left\langle \tfrac{1}{2}E_0(\hat{\nu})\right\rangle, \tag{8.3}$$

so that the other half of the dark energy $\frac{1}{2}E_{\mathrm{d}}$ is determined by all massive, sterile dark energy neutrino relics ($\hat{\nu}_{1/2}$) with the total, greatest possible, thermal energy

$$kT = \tfrac{1}{2}E_{\mathrm{d}} = \sum_{\frac{1}{2}\hat{\nu}}\left\langle kT_{\frac{1}{2}\hat{\nu}}\right\rangle = \sum_{\frac{1}{2}\hat{\nu}}\left\langle \tfrac{1}{2}E_0(\hat{\nu})\right\rangle. \tag{8.4}$$

Therefore, their gravitational potential energy $V_{\mathrm{gr}}(d_{\mathrm{eff}})$ can be defined by

$$V_{\mathrm{gr}}(d_{\mathrm{eff}}) = \frac{G_N}{c^4}\frac{(\frac{1}{2}E_{\mathrm{d}})^2}{d_{\mathrm{eff}}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2}\frac{(\frac{1}{2}E_{\mathrm{d}})^2}{d_{\mathrm{eff}}}. \tag{8.5}$$

Using $V_{\mathrm{gr}}(d_{\mathrm{eff}}) = \frac{1}{2}E_{\mathrm{d}}$, we obtain

$$d_{\mathrm{eff}} = \frac{G_N}{c^4}\tfrac{1}{2}E_{\mathrm{d}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2}\tfrac{1}{2}E_{\mathrm{d}}. \tag{8.6}$$

By Eqs. (8.1) and (8.6), we find the condition

$$d_{\mathrm{eff}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2}\tfrac{1}{2}\Omega_\Lambda\rho_{0\mathrm{C}}c^2\tfrac{4}{3}\pi\, d_{\mathrm{eff}}^3 \tag{8.7}$$

with the solution

$$d_{\mathrm{eff}} = \left(\frac{E_{\mathrm{Pl}}^2}{\hbar c\,\frac{1}{2}\Omega_\Lambda\rho_{0\mathrm{C}}c^2\,\frac{4}{3}\pi}\right)^{1/2} = 3.321\cdot 10^{26}\,\mathrm{m}, \tag{8.8}$$

so that Eq. (8.2) gives

$$r = r_{\text{eff}} = \frac{d_{\text{eff}}}{R_{\text{eff}}} = 1.2438\,. \tag{8.9}$$

Using Eqs. (8.1) and (8.8), we get quantitatively the dark energy

$$E_{\text{d}} = \Omega_{\Lambda}\rho_{0\text{C}}c^2\,\tfrac{4}{3}\pi\,d_{\text{eff}}^3 = 5.017\cdot 10^{89}\ \text{eV}\,. \tag{8.10}$$

By the result (8.10), the assumptions (8.3) and (8.4) are trivially defined by

$$kT = \tfrac{1}{2}E_{\text{d}} = \sum_{\gamma}\left\langle kT_{\gamma}\right\rangle = \sum_{\gamma}\left\langle \tfrac{1}{2}E_0(\hat{\nu})\right\rangle = 2.509\cdot 10^{89}\ \text{eV} \tag{8.11}$$

and

$$kT = \tfrac{1}{2}E_{\text{d}} = \sum_{\tfrac{1}{2}\hat{\nu}}\left\langle kT_{\tfrac{1}{2}\hat{\nu}}\right\rangle = \sum_{\tfrac{1}{2}\hat{\nu}}\left\langle \tfrac{1}{2}E_0(\hat{\nu})\right\rangle = 2.509\cdot 10^{89}\ \text{eV}\,. \tag{8.12}$$

Then, by Eq. (8.11), using the expression (3.14), the thermal photon number density $n_{\gamma}(R_{\text{eff}})$ is found to

$$n_{\gamma}(R_{\text{eff}}) = 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\tfrac{1}{2}E_{\text{d}}}{\hbar c}\right)^3 = 1.353\cdot 10^{282}\ \text{cm}^{-3}\,. \tag{8.13}$$

Because of the decay conditions of the sterile neutrinos (see Sec. 5.1), for the massive, sterile neutrino relics, their number density $n_{\tfrac{1}{2}\hat{\nu}}(R_{\text{eff}})$ must be equivalent to $n_{\gamma}(R_{\text{eff}})$, i.e. we have

$$n_{\tfrac{1}{2}\hat{\nu}}(R_{\text{eff}}) = n_{\gamma}(R_{\text{eff}}) = 1.353\cdot 10^{282}\ \text{cm}^{-3}\,. \tag{8.14}$$

At $R_{\text{eff}} = 2.67\cdot 10^{26}\,\text{m}$ (see above), in the massive universe, because of $n_{\gamma}(R_{\text{eff}}) = n_{\tfrac{1}{2}\hat{\nu}}(R_{\text{eff}})$, we have a stable equilibrium between these massless photons and this massive, sterile neutrino relics, since still no complete decay of the sterile neutrinos has taken place.

Taking the result (8.8), the lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos can be assumed to

$$\tau_{\hat{\nu}} = \frac{d_{\text{eff}}}{c} = 1.108\cdot 10^{18}\ \text{s} = 35.11\ \text{Gyr}\,. \tag{8.15}$$

Consequently, for $t_{\text{eff}} = 8.034\cdot 10^{17}\ \text{s} = 25.46\ \text{Gyr}$ (see above), if we use the denotation $n(\hat{\nu})$ for the total number density of the sterile neutrinos, because of their constant volume (see below), by the universal decay law

$$n_{\hat{\nu}}(t_{\text{eff}}) = n(\hat{\nu})\, \mathrm{e}^{-t_{\text{eff}}/\tau_{\hat{\nu}}}\,, \tag{8.16}$$

we find

$$n_{\hat{\nu}}(t_{\text{eff}}) \approx \tfrac{1}{2} n(\hat{\nu}). \tag{8.17}$$

Therefore, via Eqs. (8.13) and (8.14), we assume

$$n_{\hat{\nu}}(t_{\text{eff}}) = n_{\gamma}(R_{\text{eff}}) = n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = 1.353 \cdot 10^{282}\ \text{cm}^{-3}\,, \tag{8.18}$$

so that for the total number density of the sterile neutrinos we get

$$n(\hat{\nu}) = 2n_{\gamma}(R_{\text{eff}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) =$$

$$= 2 \times 2.701178\, \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3}\,. \tag{8.19}$$

Then, for the final state of the massive universe, at $R_{\text{f}} = 3.39 \cdot 10^{26}$ m (see Eq. 7.16)) or $t_{\text{f}} = 2.136 \cdot 10^{20}\,\text{s} = 6768\,\text{Gyr}$ (see Eq. (7.42)), we can assume a complete decay of all sterile neutrinos with $n(\hat{\nu}) = 2.706 \cdot 10^{282}\ \text{cm}^{-3}$, so that we get now correspondingly an unstable equilibrium between the massless photons with

$$n_{\gamma}(R_{\text{f}}) = 2n_{\gamma}(R_{\text{eff}}) = n(\hat{\nu}) =$$

$$= 2 \times 2.701178\, \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3} \tag{8.20}$$

and the massive, sterile neutrino relics with

$$n_{\frac{1}{2}\hat{\nu}}(R_{\text{f}}) = 2n_{\frac{1}{2}\hat{\nu}}(R_{\text{eff}}) = n(\hat{\nu}) =$$

$$= 2 \times 2.701178\, \frac{2.4041138}{\pi^2} \left(\frac{\frac{1}{2} E_{\text{d}}}{\hbar c} \right)^3 = 2.706 \cdot 10^{282}\ \text{cm}^{-3}\,. \tag{8.21}$$

This unstable equilibrium leads to a transition from the final state of the massive universe to the big bang of the massless universe.

Therefore, for example, by Eqs. (6.43) and (8.20), because of particle conservation, we have the condition

$$n_\gamma(R_{\mathrm{BB}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{BB}}^3} =$$

$$= n_\gamma(R_{\mathrm{f}}) = 2\times 2.701178\,\frac{2.4041138}{\pi^2}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^3 \tag{8.22}$$

with the solution

$$R_{\mathrm{BB}} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{1/3}\frac{\hbar c}{\frac{1}{2}E_{\mathrm{d}}} = 2.069\cdot 10^{-98}\,\mathrm{m}\,. \tag{8.23}$$

Then, the remaining parameters of the big bang are defined as follows. By Eq. (6.40), the big bang has taken place at the time

$$t_{\mathrm{BB}} = \frac{R_{\mathrm{BB}}}{c} = \left(\frac{\pi^4\,\Omega_\gamma}{292.2273}\right)^{1/3}\frac{\hbar}{\frac{1}{2}E_{\mathrm{d}}} = 6.901\cdot 10^{-107}\,\mathrm{s}\,. \tag{8.24}$$

According to Eqs. (6.37) and (6.39), for the big bang, the vacuum energy density or the cosmological "constant" are given by

$$\rho_{\mathrm{vac}}(R_{\mathrm{BB}})c^2 = \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{\mathrm{Pl}}^2}{\hbar c\,R_{\mathrm{BB}}^2} =$$

$$= \frac{2}{45}\frac{\Omega_\gamma^{1/3}}{\pi^{2/3}}(292.2273)^{2/3}\frac{E_{\mathrm{Pl}}^2}{(\hbar c)^3}(\tfrac{1}{2}E_{\mathrm{d}})^2 = 4.227\cdot 10^{247}\,\mathrm{eV\,cm^{-3}} \tag{8.25}$$

or

$$\Lambda = \frac{16}{45}\pi^3\Omega_\gamma\frac{1}{R_{\mathrm{BB}}^2} =$$

$$= \frac{16}{45}(\pi\Omega_\gamma)^{1/3}(292.2273)^{2/3}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^2 = 1.406\cdot 10^{192}\,\mathrm{m^{-2}}\,, \tag{8.26}$$

respectively.

Using Eqs. (6.34), for the big bang, the smallest possible, thermal energy kT_{BB} and the greatest possible, relativistic energy E_{BB} (uncertainty relation) are found to

$$kT_{\mathrm{BB}} = \frac{E_{\mathrm{Pl}}^2}{\hbar c} R_{\mathrm{BB}} = \left(\frac{\pi^4 \Omega_\gamma}{292.2273} \right)^{1/3} \frac{E_{\mathrm{Pl}}^2}{\frac{1}{2} E_{\mathrm{d}}} = 1.563 \cdot 10^{-35}\ \mathrm{eV} \tag{8.27}$$

and

$$E_{\mathrm{BB}} = \frac{\hbar c}{R_{\mathrm{BB}}} = \frac{E_{\mathrm{Pl}}^2}{kT_{\mathrm{BB}}} = 9.537 \cdot 10^{90}\ \mathrm{eV}\,. \tag{8.28}$$

The expression $\hbar c / R_{\mathrm{BB}}$ is interpreted as the potential energy of an attractive force, similar to the gravitational force. To this new force, we will return in Eqs. (8.117) and (8.118).

With that, we have solved the problem "big bang". The transition from the final state of the massive universe ($R_{\mathrm{f}} \geq R \geq R_{\mathrm{Pl}}$) to the big bang of the massless universe ($0 \rightarrow R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) {with zero-point oscillations (see Sec. 8.4)} leads to a cyclic evolution of the total [massless ($R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$) and massive ($R_{\mathrm{f}} \geq R \geq R_{\mathrm{Pl}}$)] universe.

Using the data of Tables III to V, all results, derived in this Sec. 8.1 for the total universe, are also valid for the total (massless and massive) anti-universe and its cyclic evolution.

8.2 The lifetime of the sterile neutrinos in the total universe

According to Ref. [1], in this chapter, we estimate the lifetime of the sterile neutrinos in the total universe. For this goal, we must still determine the initial, sterile neutrino number density.

Therefore, we use the fact that the present day universe contains a relic neutrino (ν) background with the temperature $T_{\nu,0} = (4/11)^{1/3} T_0 = 1.945\,\mathrm{K}$ (see Eq. (2.10)), where ν characterizes the light neutrinos, which were assumed as massless or nearly massless.

For the formation of the particle-defined cosmological parameters (see Refs. [1-3]), which were identified as the cosmological parameters of the heavy neutrinos (see Sec. 4.2), the necessary energy for the light neutrinos was taken

from the early universe, so that the light and the heavy neutrinos have the same number density, i.e. $n(\nu) = n(\tilde{\nu}) = 112\,\mathrm{cm}^{-3}$ (see Eq. (3.24) and Sec. 4).

Consequently, the light (ν) and the heavy ($\tilde{\nu}$) neutrino background must possess the same temperature $T_{\nu,0} = T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$ according to the ideal gas law because of the constant pressure.

At the pressure P, between the energy E and the volume V, for the expansion of the massive universe, the kinetic theory of gases yields

$$dE = -PdV. \tag{8.29}$$

At the formation of sterile neutrinos, the necessary energy is taken from the relic neutrino ($\tilde{\nu}$) background, whereat the number density of the sterile neutrinos ($\hat{\nu}$) decreases to $n(\hat{\nu}) = 0.178\,\mathrm{cm}^{-3}$ (see Sec. 5.1).

Therefore, according to Eq. (8.29), at constant pressure, for the heavy neutrino relic of $T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$, we must here assume the small volume changing $-1/n(\hat{\nu})$, so that we get a cooling of this relic neutrino background from $T_{\tilde{\nu},0} = 1.945\,\mathrm{K}$ to $T_{\hat{\nu},0}$ in the large volume changing $-1/n(\tilde{\nu})$, so that the ideal gas law yields

$$T_{\hat{\nu},0} = \frac{n(\hat{\nu})}{n(\tilde{\nu})} T_{\tilde{\nu},0} = 3.09 \cdot 10^{-3}\,\mathrm{K}, \tag{8.30}$$

i.e. the corresponding thermal photon energy is given by

$$kT = k(11/4)^{1/3} T_{\hat{\nu},0} = 3.73 \cdot 10^{-7}\,\mathrm{eV}. \tag{8.31}$$

Then, using Eqs. (6.34) and (8.31), for the quantum gravity of the massless universe ($R \le R_{\mathrm{Pl}}$), we obtain the distance

$$R = R_{\hat{\nu}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2} k(11/4)^{1/3} T_{\hat{\nu},0} = 4.938 \cdot 10^{-68}\,\mathrm{cm}, \tag{8.32}$$

so that Eqs. (6.41) or (6.43) yield the initial, thermal number density of the sterile neutrinos via the massive relics or the massless photons from the decay of the sterile neutrinos in the massless universe to

$$n_{\hat{\nu}}(R_{\hat{\nu}}) = n_{\gamma}(R_{\hat{\nu}}) = \frac{2}{3}\Omega_{\gamma}\frac{\pi^2}{15}\frac{1}{R_{\hat{\nu}}^3} = 1.989 \cdot 10^{197}\,\mathrm{cm}^{-3}. \tag{8.33}$$

Therefore, by the universal decay law, for $n_\gamma(R_{\hat{\nu}}) = 1.989 \cdot 10^{197}\,\mathrm{cm}^{-3}$ (see Eq. (8.33)) and $n_\gamma(R_\mathrm{f}) = 2.706 \cdot 10^{282}\,\mathrm{cm}^{-3}$ (see Eq. (8.20)), using a constant sterile neutrino volume $V(\hat{\nu})$ for $n_\gamma(R_{\hat{\nu}})$ and $n_\gamma(R_\mathrm{f})$, we can assume

$$n_\gamma(R_{\hat{\nu}}) = n_\gamma(R_\mathrm{f})\,\mathrm{e}^{-t_\mathrm{f}/\tau_{\hat{\nu}}}\,, \tag{8.34}$$

where t_f is the age of the final state of the massive universe, whereas $\tau_{\hat{\nu}}$ is the lifetime of the sterile neutrinos.

Their constant volume $V(\hat{\nu})$ is determined in Sec. 8.3. Now, we have two possibilities.

Firstly, in the variant 1, we can thus calculate $\tau_{\hat{\nu}} = \tau_{\hat{\nu}1}$, using the age $t_\mathrm{f} = t_{\mathrm{f}1} = 2.136 \cdot 10^{20}\,\mathrm{s} = 6768\,\mathrm{Gyr}$ (see Eq. (7.42)) for the final state of the massive universe.

Secondly, in the variant 2, we can estimate the age $t_\mathrm{f} = t_{\mathrm{f}2}$ for the final state of the massive universe, taking $\tau_{\hat{\nu}} = \tau_{\hat{\nu}2} = 1.108 \cdot 10^{18}\,\mathrm{s} = 35.11\,\mathrm{Gyr}$ (see Eq. (8.15)).

Then, taking the results (8.20) and (8.33), via Eq. (8.34), the variant 1 yields

$$\tau_{\hat{\nu}1} = t_{\mathrm{f}1}\left(\ln\left\{n_\gamma(R_\mathrm{f})/n_\gamma(R_{\hat{\nu}})\right\}\right)^{-1} = 1.090 \cdot 10^{18}\,\mathrm{s} = 34.52\,\mathrm{Gyr}\,, \tag{8.35}$$

whereas the variant 2 provides

$$t_{\mathrm{f}2} = \tau_{\hat{\nu}2}\ln\left\{n_\gamma(R_\mathrm{f})/n_\gamma(R_{\hat{\nu}})\right\} = 2.172 \cdot 10^{20}\,\mathrm{s} = 6883\,\mathrm{Gyr}\,. \tag{8.36}$$

Now, we show that the variant 2 is correct. For this goal, we introduce the distances

$$d_{\mathrm{f}1} = R_\mathrm{f} r_{\mathrm{f}1} = c t_{\mathrm{f}1} = 6.404 \cdot 10^{28}\,\mathrm{m} \tag{8.37}$$

and

$$d_{\mathrm{f}2} = R_\mathrm{f} r_{\mathrm{f}2} = c t_{\mathrm{f}2} = 6.512 \cdot 10^{28}\,\mathrm{m}\,, \tag{8.38}$$

where $r_{\mathrm{f}1}$ and $r_{\mathrm{f}2}$ describe again the dimensionless, time-independent, comoving coordinate distances r (see Eqs. (8.2) and (8.9)), i.e. because of the dark energy conservation from Eq. (8.10) we can form the energy densities

$$\rho_{\mathrm{f}1} c^2 = \Omega_\Lambda \rho_{0\mathrm{C}} c^2 \left(\frac{d_\mathrm{eff}}{d_{\mathrm{f}1}}\right)^3 = 4.560 \cdot 10^{-4}\,\mathrm{eV\,cm}^{-3} \tag{8.39}$$

and

$$\rho_{f2}c^2 = \Omega_\Lambda \rho_{0C} c^2 \left(\frac{d_{\text{eff}}}{d_{f2}}\right)^3 = 4.337 \cdot 10^{-4}\,\text{eV}\,\text{cm}^{-3}, \tag{8.40}$$

so that we have the ratios

$$\frac{\rho_{f1}}{\rho_{f2}} = \frac{V_{f2}}{V_{f1}} = \left(\frac{d_{f2}}{d_{f1}}\right)^3 = \left(\frac{t_{f2}}{t_{f1}}\right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}}\right)^3 \cong 1.052. \tag{8.41}$$

The discrepancy between $t_f = t_{f1} = 2.136 \cdot 10^{20}\,\text{s} = 6768\,\text{Gyr}$ (see Eq. (7.42)) and $t_{f2} = 2.172 \cdot 10^{20}\,\text{s} = 6883\,\text{Gyr}$ (see Eq. (8.36)) has the plausible reason that the derivation of $t_f = t_{f1}$ is alone based on the vacuum energy density $\rho_{\text{vac}} c^2 = \Omega_\Lambda \rho_{0C} c^2 = 3.27 \cdot 10^3\,\text{eV}\,\text{cm}^{-3}$ (see Eq. (6.48)) or the cosmological "constant" $\Lambda = 3\Omega_\Lambda / R_0^2 = 1.087 \cdot 10^{-52}\,\text{m}^{-2}$ (see Eq. (6.51)), i.e. the limiting conditions (6.56) and (6.57) yield here the temperature $T = T_{f1} = 51.41\,\text{K}$ (see Eq. (6.58)).

However, this discrepancy can be explained uniquely by Eq. (6.66), using the assumptions for the times $\tilde{t} = \tau_{\hat{\nu}2} \ln 2 = 7.680 \cdot 10^{17}\,\text{s}$ ($\tau_{\hat{\nu}2}$ see Eqs. (8.15)) and $t = t_{\text{eff}} = 8.034 \cdot 10^{17}\,\text{s}$ (see Eq. (7.36)), so that Eq. (6.66) provides

$$\left(\frac{c^2 \tilde{\Lambda}}{3}\right)^{1/2} = \frac{t_{\text{eff}} - t_0}{(\tau_{\hat{\nu}2} \ln 2) - t_0} \Omega_\Lambda^{1/2} H_0 = 1.997 \cdot 10^{-18}\,\text{s}^{-1}. \tag{8.42}$$

By Eq. (8.42), we obtain

$$\tilde{\Lambda} = 1.331 \cdot 10^{-52}\,\text{m}^{-2}, \tag{8.43}$$

so that the temperature $T = T_{f2} = 54.07\,\text{K}$ results from Eqs. (6.53) and (8.43) for $N(T) = 3.362644$ (see Eq. (2.9) or Refs. [5, 7]).

For the constant pressure $P = -\Omega_\Lambda \rho_{0C} c^2$, the ideal gas law yields following connection between the temperatures $T = T_{f2} = 54.07\,\text{K}$ (see above) and $T = T_{f1} = 51.41\,\text{K}$ (see above) as well as the corresponding volumes V_{f2} and V_{f1} for the state changing according to Eq. (8.41):

$$\frac{T_{f2}}{T_{f1}} = \frac{V_{f2}}{V_{f1}} = \left(\frac{d_{f2}}{d_{f1}}\right)^3 = \left(\frac{t_{f2}}{t_{f1}}\right)^3 = \left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}}\right)^3 \cong 1.052\,. \tag{8.44}$$

Thus, we get

$$t_{f2} = t_{f1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 6883\ \text{Gyr} \tag{8.45}$$

and

$$\tau_{\hat{\nu}2} = \tau_{\hat{\nu}1}\left(\frac{T_{f2}}{T_{f1}}\right)^{1/3} = 35.11\ \text{Gyr}, \tag{8.46}$$

i.e. $t_{f2} = 6883\,\text{Gyr}$ (see Eq. (8.36)) and $\tau_{\hat{\nu}2} = 35.11\,\text{Gyr}$ (see Eq. (8.15)) are correct. Because of Eqs. (8.35) and (8.36), the results (8.45) and (8.46) provide the evidence, that for $R \le R_{Pl}$ the assumptions $\text{const} = 1/N(T)$ and $N(T) = 1/2\Omega_\gamma$ (see Eqs. (6.35) to (6.38)) are justified.

Using the data of Tables III to V, the results, derived in this Sec. 8.2 for the total universe, are also valid for the total anti-universe.

8.3 The constant volume of the sterile neutrinos

In this chapter, we derive the constant volume $V(\hat{\nu})$ of the sterile neutrinos (see Sec. 8.2) in the total universe.

For this goal, we assume that because of the energy conservation in the final state of the massive universe by the dark energy $E_d = 5.017 \cdot 10^{89}\,\text{eV}$ (see Eq. (8.10)) the volume V_f is occupied, i.e. we have

$$V_f = \frac{E_d}{\Omega_\Lambda \rho_{0C} c^2} = 1.534 \cdot 10^{86}\ \text{cm}^3\,. \tag{8.47}$$

Using the number density $n(\hat{\nu}) = 0.178\,\text{cm}^{-3}$ of the sterile neutrinos (see Sec. 8.2), we can introduce their constant volume $V(\hat{\nu})$ to

$$V(\hat{\nu}) = \frac{\text{const}(\hat{\nu})}{n(\hat{\nu})}, \tag{8.48}$$

so that for the final state of the massive universe the number N_{f} of the sterile neutrinos is given by

$$N_{\text{f}} = \frac{V_{\text{f}}}{V(\hat{\nu})} = \frac{n(\hat{\nu})\, V_{\text{f}}}{\text{const}(\hat{\nu})} = \frac{2.731 \cdot 10^{85}}{\text{const}(\hat{\nu})}. \tag{8.49}$$

At the enormous age of the final state of the massive universe (see Sec. 8.2), we have assumed a complete decay of the N_{f} sterile neutrinos.

Consequently, in the massless universe, the beginning of the sterile neutrino decay, which must lead to the results (8.30) to (8.33), takes place in one massless photon and one (massive) sterile neutrino relic, so that instead of the volume V_{f} these 2 different decay products must yield the initial (i) volume of the sterile neutrinos in the massless universe to

$$V_{\text{i}} = \frac{2}{n(\hat{\nu})} = 11.24\,\text{cm}^3. \tag{8.50}$$

Thus, the initial number N_{i} of the sterile neutrinos of the massless universe (see Eq. (8.33)) is defined by

$$N_{\text{i}} = \frac{V_{\text{i}}}{V(\hat{\nu})} = \frac{2}{\text{const}(\hat{\nu})}. \tag{8.51}$$

Therefore, instead of Eq. (8.34), we can assume

$$N_{\text{i}} = N_{\text{f}}\, \text{e}^{-t_{\text{f}}/\tau_{\hat{\nu}}}, \tag{8.52}$$

so that via the Eqs. (8.48) to (8.52) we can form the expressions

$$N_{\text{f}} = V(\hat{\nu})\, n_{\gamma}(R_{\text{f}}) = \frac{n(\hat{\nu})\, V_{\text{f}}}{\text{const}(\hat{\nu})} = \frac{\text{const}(\hat{\nu})}{n(\hat{\nu})}\, n_{\gamma}(R_{\text{f}}) \tag{8.53}$$

and

$$N_{\text{i}} = V(\hat{\nu})\, n_{\gamma}(R_{\hat{\nu}}) = \frac{2}{\text{const}(\hat{\nu})} = \frac{\text{const}(\hat{\nu})}{n(\hat{\nu})}\, n_{\gamma}(R_{\hat{\nu}}). \tag{8.54}$$

Then, the expressions (8.53) and (8.54) provide the conditions

$$\frac{n(\hat{\nu})\, V_{\text{f}}}{\text{const}(\hat{\nu})} = \frac{\text{const}(\hat{\nu})}{n(\hat{\nu})}\, n_{\gamma}(R_{\text{f}}) \tag{8.55}$$

and

$$\frac{2}{\mathrm{const}(\hat{\nu})} = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})} n_\gamma(R_{\hat{\nu}}) . \tag{8.56}$$

Thus, the conditions (8.55) or (8.56) yield

$$\mathrm{const}(\hat{\nu}) = \left(\frac{n^2(\hat{\nu})\, V_\mathrm{f}}{n_\gamma(R_\mathrm{f})} \right)^{1/2} \cong 1.34 \cdot 10^{-99} \tag{8.57}$$

or

$$\mathrm{const}(\hat{\nu}) = \left(\frac{2\, n(\hat{\nu})}{n_\gamma(R_{\hat{\nu}})} \right)^{1/2} \cong 1.34 \cdot 10^{-99} , \tag{8.58}$$

so that Eq. (8.48) leads to the constant, sterile neutrino volume

$$V(\hat{\nu}) = \frac{\mathrm{const}(\hat{\nu})}{n(\hat{\nu})} \cong 7.528 \cdot 10^{-99}\ \mathrm{cm}^3 . \tag{8.59}$$

Taking Eqs. (8.20) and (8.33) as well as (8.53) to (8.59), we obtain the particle numbers (see above):

$$N_\mathrm{f} = V(\hat{\nu})\, n_\gamma(R_\mathrm{f}) \cong 2.04 \cdot 10^{184} \tag{8.60 a}$$

and

$$N_\mathrm{i} = V(\hat{\nu})\, n_\gamma(R_{\hat{\nu}}) \cong 1.50 \cdot 10^{99} . \tag{8.60 b}$$

Consequently, by Eqs. (8.20) and (8.33), considering the expressions (8.47) as well as (8.49 to (8.51)), the results (8.60 a) and (8.60 b) yield the following connections for the constant volume $V(\hat{\nu})$ of the sterile neutrinos

$$V(\hat{\nu}) = \frac{V_\mathrm{f}}{N_\mathrm{f}} = \frac{N_\mathrm{f}}{n_\gamma(R_\mathrm{f})} \cong 7.53 \cdot 10^{-99}\ \mathrm{cm}^3 \tag{8.61 a}$$

or

$$V(\hat{\nu}) = \frac{V_\mathrm{i}}{N_\mathrm{i}} = \frac{N_\mathrm{i}}{n_\gamma(R_{\hat{\nu}})} \cong 7.53 \cdot 10^{-99}\ \mathrm{cm}^3 \tag{8.61 b}$$

in accordance with Eq. (8.59).

Thus, the expressions (8.60 a) to (8.61 b) confirm the results (8.35) and (8.36) because of Eq. (8.34), i.e. we have again

$$\frac{t_{\mathrm{f}}}{\tau_{\hat{\nu}}} = \ln \frac{n_{\gamma}(R_{\mathrm{f}})}{n_{\gamma}(R_{\hat{\nu}})} = \ln \frac{N_{\mathrm{f}}}{N_{\mathrm{i}}} \cong 196.03 . \tag{8.62}$$

With that, the assumption (8.34) is proved.

Using the data of Tables III to V, the results, derived in this Sec. 8.3 for the total universe, are also valid for the total anti-universe.

8.4 Zero-point oscillations as origin of the massless universes

In this chapter, we explain the high number densities (8.20) and (8.21) for the decay products of the sterile neutrinos as well as the zero-point oscillations as origin of the massless universes.

For the first goal, we use the expression (8.29), where normally the "$-$"sign defines an energy decrease at the expansion of the massive universe. However, if for the final state of the massive universe the connection $-P = \rho_{\mathrm{vac}} c^2$ is assumed, we obtain a gigantic energy increase, which by heat accumulation leads to an overheat, so that the consequence is a heat death of this final state of the massive universe because of the accumulation of the decay products of the sterile neutrinos, since the vacuum energy density $\rho_{\mathrm{vac}} c^2$ (see Eq. (6.52)) can be attributed to the half dark energy $\frac{1}{2} E_{\mathrm{d}}$ (see Eqs. (8.10) to (8.12)). This fact confirms the hypothesis of a joint origin of the dark matter and the dark energy by sterile neutrinos (see Sec. 5). Up to this time, in contrast to this heat death, a cold death of the massive universe was assumed. Because of the expressions (8.19) to (8.21), in the final state of the massive universe, the total energy density of the sterile neutrinos can be described by $\rho(\hat{\nu}) c^2 = \rho_{\gamma}(R_{\mathrm{f}}) c^2 + \rho_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}}) c^2$. Then, using Eqs. (3.13) and (6.52), in the final state of the massive universe, for its energy density $\rho_{\gamma}(R_{\mathrm{f}}) c^2$ or $\rho_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}})$ of every type of the decay products of the sterile neutrinos, we can assume

$$2\frac{\pi^2}{15}\frac{(\frac{1}{2}E_{\mathrm{d}})^4}{(\hbar c)^3} = \rho_\gamma(R_{\mathrm{f}})\,c^2 = \rho_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}})\,c^2 = \tfrac{1}{2}\rho(\hat{\nu})\,c^2 = 2\,\rho_{\mathrm{vac}}\,c^2 =$$

$$= 2\frac{1}{3}\frac{\pi^2}{15}\frac{(kT_1)^4}{(\hbar c)^3}\frac{1}{N(T)}, \tag{8.63}$$

so that we obtain the heat accumulation

$$kT_1 = [3\,N(T)]^{\frac{1}{4}}\,\tfrac{1}{2}E_{\mathrm{d}}, \tag{8.64}$$

which leads to an overheat of this final state by the very high energy density (8.63), i.e. this massive universe meets one's heat death. Consequently, via $E_{\mathrm{K}}(\gamma) = 2.701178 \times \frac{1}{2}E_{\mathrm{d}}$ (see Eq. (3.21)), in accordance with Eqs. (8.11) to (8.14) as well as (8.19) to (8.21), this energy density (8.63) is thus converted into the thermal number density of the accumulation of the decay products of the sterile neutrinos

$$n_\gamma(R_{\mathrm{f}}) = 2.701178\,\frac{\rho_\gamma(R_{\mathrm{f}})\,c^2}{E_{\mathrm{K}}(\gamma)} =$$

$$= n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}}) = 2.701178\,\frac{\rho_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}})\,c^2}{E_{\mathrm{K}}(\gamma)} = 2\frac{\pi^2}{15}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^3. \tag{8.65}$$

Similar to Eqs. (8.63) and (8.65), taking Eqs. (3.21) as well as (8.19) to (8.21), we can express the energy density $\frac{1}{2}\rho(\hat{\nu})\,c^2 = \rho_\gamma(R_{\mathrm{f}})\,c^2 = \rho_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}})\,c^2$ of the sterile neutrinos also as follows

$$\frac{\pi^2}{15}\frac{(kT_2)^4}{(\hbar c)^3} = \tfrac{1}{2}\rho(\hat{\nu})\,c^2 = 2\frac{\pi^2}{15}\frac{(\frac{1}{2}E_{\mathrm{d}})^4}{(\hbar c)^3}, \tag{8.66}$$

so that we get the heat accumulation

$$kT_2 = 2^{\frac{1}{4}}\,\tfrac{1}{2}E_{\mathrm{d}}, \tag{8.67}$$

i.e. analogous to Eq. (8.65) we find

$$n(\hat{\nu}) = 2.701178\,\frac{\frac{1}{2}\rho(\hat{\nu})\,c^2}{E_{\mathrm{K}}(\gamma)} =$$

$$= 2\,\frac{\pi^2}{15}\left(\frac{\frac{1}{2}E_{\mathrm{d}}}{\hbar c}\right)^3 . \tag{8.68}$$

Then, by Eqs (8.64) and (8.67), we can introduce the ratio

$$\frac{T_1}{T_2} = \left(\frac{3\,N(T)}{2}\right)^{1/4} , \tag{8.69}$$

so that for the final state of the massive universe the entropy conservation (see, e.g., Refs. [5, 7]) yields

$$\left(\frac{T_1}{T_2}\right)^3 = \left(\frac{3\,N(T)}{2}\right)^{3/4} = \frac{V_{\mathrm{f}2}}{V_{\mathrm{f}1}} = \left(\frac{d_{\mathrm{f}2}}{d_{\mathrm{f}1}}\right)^3 . \tag{8.70}$$

Using the condition (8.41) in the result (8.70), we obain

$$\left(\frac{3\,N(T)}{2}\right)^{3/4} \cong 1.052 \tag{8.71}$$

with the solution

$$N(T) \cong 0.713 \tag{8.72}$$

in accordance with

$$N(T) = \left(\frac{4}{11}\right)^{1/3} \cong 0.713 \quad \text{(see Eq. (2.10))} . \tag{8.73}$$

By the conditions (8.41) as well as (8.70) to (8.73), the results (8.45) and (8.46) are confirmed because of

$$t_{\mathrm{f}2} = t_{\mathrm{f}1}\left(\frac{3\,N(T)}{2}\right)^{1/4} \cong 6883\ \mathrm{Gyr} \tag{8.74}$$

and

$$\tau_{\hat{\nu}2} = \tau_{\hat{\nu}1}\left(\frac{3\,N(T)}{2}\right)^{1/4} \cong 35.11\ \mathrm{Gyr} . \tag{8.75}$$

With that, the conception of a heat death of the final state of the massive universe, which simultaneously means the birth of the massless universe by the big bang, is proved via the corresponding vacuum energy density (see Eq. (6.52)) of the quantum gravity.

Then, in other words, by the quantities (8.23) and (8.28) as well as (8.30) and (8.32), via Eqs. (8.74) and (8.75), we must obtain the connections

$$\frac{T_{\hat{\nu},0}}{T_{\mathrm{BB}}}\frac{R_{\mathrm{BB}}}{R_{\hat{\nu}}}=\frac{2}{3}\left(\frac{t_{\mathrm{f}2}}{t_{\mathrm{f}1}}\right)^4=N(T)=\left(\frac{4}{11}\right)^{1/3} \tag{8.76}$$

and

$$\frac{T_{\hat{\nu},0}}{T_{\mathrm{BB}}}\frac{R_{\mathrm{BB}}}{R_{\hat{\nu}}}=\frac{2}{3}\left(\frac{\tau_{\hat{\nu}2}}{\tau_{\hat{\nu}1}}\right)^4=N(T)=\left(\frac{4}{11}\right)^{1/3}. \tag{8.77}$$

Considering the results (3.8) and (3.34) as well as (3.45) and (5.29), the expression (8.31) can be confirmed semi-empirically by the photon energy

$$kT=\Omega_{\tilde{\nu}}(\hat{\nu}_{\mathrm{CMB}})\,\alpha_{\mathrm{GUT}}\,E_0(\nu_{\mathrm{e}})\cong 3.73\cdot 10^{-7}\,\mathrm{eV}, \tag{8.78}$$

i.e. instead of Eq. (8.30) via the neutrino temperature $T_\nu=(4/11)^{1/3}T$ (see Eqs. (2.10) and (8.78)) we find also the condition

$$\begin{aligned}T_{\hat{\nu},0}\to T_{\hat{\nu}_{\mathrm{CMB}}}&=(4/11)^{1/3}\,\Omega_\nu(\tilde{\nu}_{\mathrm{CMB}})\,\alpha_{\mathrm{GUT}}\,E_0(\nu_{\mathrm{e}})/k=\\&=(4/11)^{1/3}\,3.7297\cdot 10^{-5}E_0(\hat{\nu}_{\mathrm{CMB}})\,\alpha_{\mathrm{GUT}}\,\frac{E_0(\nu_{\mathrm{e}})/k}{\mathrm{eV}}=\\&=(4/11)^{1/3}\,6.02\cdot 10^{-3}\alpha_{\mathrm{GUT}}\,E_0(\nu_{\mathrm{e}})/k\cong 3.09\cdot 10^{-3}\,\mathrm{K},\end{aligned} \tag{8.79}$$

so that the temperature condition $T_{\hat{\nu},0}\to T_{\hat{\nu}_{\mathrm{CMB}}}$ (see Eq. (8.79)) supports the interpretations (8.76) and (8.77).

Because the overheat of the final state of the massive universe means its heat death and simultaneously the corresponding birth of the quantum gravity by the big bang (see Sec. 8.1) as the beginning of the massless universe. Here, the decay products of the sterile neutrinos (because of their dark energy parts according to Eqs. (8.11) and (8.12)) produce zero-point oscillations, which are responsible for the origin of the massless universe (see below).

The particle formation of the massless universe up to its boundary $R = R_{\mathrm{Pl}}$ (see Secs. 6.2 and 6.3) is the reason that via its corresponding, mean energy densities $E_{\mathrm{Pl}}\, n_{\gamma}(R_{\mathrm{f}})$ or $E_{\mathrm{Pl}}\, n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}})$ according to Eqs. (6.20) as well as (8.20) and (8.21) we can prove the energy conservation.

Consequently, for example, because of the joint boundary $R = R_{\mathrm{Pl}}$, in the massless and massive universe, we can assume as the two mean energy densities of the decay products of the sterile neutrinos

$$\overline{\rho}_1\, c^2 = E_{\mathrm{Pl}}\, n_{\gamma}(R_{\mathrm{f}}) = 3.304 \cdot 10^{310}\ \mathrm{eV\,cm^{-3}} \tag{8.80}$$

or

$$\overline{\rho}_1\, c^2 = E_{\mathrm{Pl}}\, n_{\frac{1}{2}\hat{\nu}}(R_{\mathrm{f}}) = 3.304 \cdot 10^{310}\ \mathrm{eV\,cm^{-3}}\,. \tag{8.81}$$

Via the sterile neutrino volume $V(\hat{\nu}) = 7.528 \cdot 10^{-99}\ \mathrm{cm}^3$ (see Eq. (8.59)), because of the transition from the final state of the universe to the big bang as the birth of the massless universe via the complete decay of the sterile neutrinos (see Sec. 8.1), we can evaluate correspondingly the total, mean energy

$$\overline{E}_1 = \overline{\rho}_1\, c^2 \times V(\hat{\nu}) \cong 2.49 \cdot 10^{212}\ \mathrm{eV}\,. \tag{8.82}$$

Then, using the sterile neutrino number $N_{\mathrm{f}} = 2.04 \cdot 10^{184}$ (see Eq. (8.60 a)), the greatest possible, relativistic particle energy (Planck energy (see Secs. 6.2 and 6.3)) is defined by

$$E_{\mathrm{Pl}} = \frac{\overline{E}_1}{N_{\mathrm{f}}} \cong 1.22 \cdot 10^{28}\ \mathrm{eV}\,. \tag{8.83}$$

For the calculation of the results (8.82) and (8.83), a second possibility is given as follows. Taking the expressions (8.63) and (8.66) [as well as (8.70) and (8.73)], via the ratio $4\rho_{\mathrm{vac}}/\rho(\hat{\nu})$, multiplied by $\rho(\hat{\nu})\, c^2 \times (T_2/T_1)^4$, for the final state of the massive universe, we can derive the energy density

$$\rho_{\mathrm{f}}(\hat{\nu})\, c^2 = \rho(\hat{\nu})\, c^2 \times \left(\frac{T_2}{T_1}\right)^4 = 4\rho_{\mathrm{vac}}\, c^2 \times \left(\frac{T_2}{T_1}\right)^4 =$$

$$= 4\,\frac{\pi^2}{15}\,\frac{(\frac{1}{2}E_{\mathrm{d}})^4}{(\hbar c)^3}\,\frac{2}{3\,N(T)} \cong 1.268 \cdot 10^{372}\ \mathrm{eV\,cm^{-3}}\,. \tag{8.84}$$

Using the transition from the final state of the massive universe to the big bang by the complete decay of the sterile neutrinos (see Sec. 8.1), analogous to the result (8.84), the vacuum energy density (8.25) permits also the introduction of the energy density

$$\rho_{\mathrm{BB}}(\hat{\nu})\,c^2 = 4\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2 \times \left(\frac{T_2}{T_1}\right)^4 =$$

$$= 4\,\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2\,\frac{2}{3\,N(T)} \cong 1.579\cdot 10^{248}\,\mathrm{eV\,cm^{-3}}\,. \tag{8.85}$$

Then, for the joint boundary $R = R_{\mathrm{Pl}}$, in the massless and massive universe, we can estimate correspondingly the mean energy density of the sterile neutrinos to

$$\overline{\rho}_2\,c^2 = (\rho_{\mathrm{f}}(\hat{\nu})\,c^2 \times \rho_{\mathrm{BB}}(\hat{\nu})\,c^2)^{1/2} \cong 1.415\cdot 10^{310}\,\mathrm{eV\,cm^{-3}}\,, \tag{8.86}$$

so that via the volume $V = \frac{4}{3}\pi R_{\mathrm{Pl}}^3 = 1.768\cdot 10^{-98}\,\mathrm{cm}^3$ the massless and massive universe yield again the total, mean energy

$$\overline{E}_2 = \overline{\rho}_2\,c^2 \times V \cong 2.50\cdot 10^{212}\,\mathrm{eV}\,, \tag{8.87}$$

i.e. because of the sterile neutrino number $N_{\mathrm{f}} = 2.04\cdot 10^{184}$ (see Eq. (8.60 a)) we obtain again the greatest possible, relativistic particle energy (Planck energy (see Secs. 6.2 and 6.3)):

$$E_{\mathrm{Pl}} = \frac{\overline{E}_2}{N_{\mathrm{f}}} \cong 1.22\cdot 10^{28}\,\mathrm{eV}\,. \tag{8.88}$$

Because we have used the number densities $n_\gamma(R_{\mathrm{f}})$ and $n_{1/2\hat{\nu}}(R_{\mathrm{f}})$ of the decay products of the sterile neutrinos for the expressions (8.80) and (8.81), we must take the sterile neutrino volume $V(\hat{\nu}) = 7.528\cdot 10^{-99}\,\mathrm{cm}^3$ (see above) in Eq. (8.82). However, because of the result (8.86), i.e. the average of the two energy densities (8.84) and (8.85), we must consider the volume $V = \frac{4}{3}\pi R_{\mathrm{Pl}}^3$ (see above) for the mean energy (8.87).

It is plausible that the mean energies (8.82) and (8.87) must be identical, since the ratio of the mean densities $\overline{\rho}_1$ (see Eqs. (8.80) and (8.81)) and $\overline{\rho}_2$ (see Eq. (8.86)) is defined by

$$\frac{\bar{\rho}_1}{\bar{\rho}_2} = \frac{V}{V(\hat{\nu})} \cong 2.34 . \tag{8.89}$$

Thus, the results (8.82) and (8.83) as well as (8.87) and (8.88) show clearly that the energy conservation is fulfilled between the massless and massive universe, whereat according to Eqs. (8.83) and (8.88) the basis of the total, mean energies (8.82) and (8.87) for these universes is the Planck energy as greatest possible particle energy.

All these considerations demonstrate uniquely that the above-mentioned zero-point oscillations must lead to the origin of the massless universes, i.e., for example, via the corresponding distance range $R_{\mathrm{BB}} \leq R \leq R_{\mathrm{Pl}}$ the zero-point oscillations define the beginning $R = R_{\mathrm{BB}}$ of the massless universe.

In other words, at the distance $R_{\mathrm{BB}} = 2.069 \cdot 10^{-96}$ cm (see Eq. (8.23)), these zero-point oscillations must be formed via the greatest possible, relativistic energy $E_{\mathrm{BB}} = \hbar c / R_{\mathrm{BB}} = 9.537 \cdot 10^{90}$eV (see Eq. (8.28)).

Now, we prove the formation of these zero-point oscillations at the joint origin of the massless universe and anti-universe. For this goal, we use de Broglie waves. In the extremely relativistic case, between its momentum p of the particle as well as its relativistic energy E, we have the connection

$$p = \frac{E}{c} . \tag{8.90}$$

Then, the de Broglie wavelength λ of the sterile neutrinos, which decay by the gravitation (see Secs. 5.1 and 8.1) is defined by

$$\lambda = 2\pi \frac{\hbar}{p} = \frac{\hbar c}{E/2\pi} . \tag{8.91}$$

For Eq. (8.91), we can write

$$\frac{\hbar c}{\lambda} = \frac{E}{2\pi} . \tag{8.92}$$

Because we have $N_{\hat{\nu}\uparrow} = 3$ excited, sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$, which describe the dark energy (see Sec. 5 as well as Eqs. (8.10) to (8.12)), we can introduce $E(\hat{\nu}_\Lambda) + E(\hat{\nu}_{\mathrm{dm}}) + E(\hat{\nu}_{\mathrm{b}}) \to E$ for $E(\hat{\nu}_\Lambda) = E(\hat{\nu}_{\mathrm{dm}}) = E(\hat{\nu}_{\mathrm{b}})$ in Eqs. (8.91) and (8.92), so that analogously we obtain also the condition

$\hbar c/\lambda(\hat{\nu}_\Lambda) = \hbar c/\lambda(\hat{\nu}_{\mathrm{dm}}) = \hbar c/\lambda(\hat{\nu}_{\mathrm{b}})$ with $\lambda(\hat{\nu}_\Lambda) = \lambda(\hat{\nu}_{\mathrm{dm}}) = \lambda(\hat{\nu}_{\mathrm{b}}) \to \lambda$. The arrow $\uparrow$ describes their spin direction.

Analogous to the Dirac particle-antiparticle theory, the zero-point oscillations must excite simultaneously $N_{\hat{\bar{\nu}}\uparrow} = 3$ sterile anti-neutrinos $\hat{\bar{\nu}}$ in the massless anti-universe, so that we obtain $N = 3$ sterile neutrino-antineutrino pairs.

However, if we consider correspondingly still $N_{\hat{\nu}\downarrow} = 3$ sterile neutrinos and $N_{\hat{\bar{\nu}}\downarrow} = 3$ with opposite spin direction (Pauli exclusions principle), we have together $N = 6$ excited sterile neutrino-antineutrino pairs.

By doubling of the $N = 6$ sterile neutrino-antineutrino pairs, we get $N = 12$ excited sterile neutrino-antineutrino pairs.

Then, for the case of N excited sterile neutrino-antineutrino pairs, it is possible to introduce the real excitation (exc) energy $E_{\mathrm{exc}}(N) = E_{\mathrm{BB}}$, where $E_{\mathrm{BB}} \equiv \hbar c/R_{\mathrm{BB}} = 9.537 \cdot 10^{90}$ eV (see Eq. (8.28)). Thus, at N excited sterile neutrino-antineutrino pairs, by the transition $E \to E_{\mathrm{exc}}(N)$, Eq. (8.92) can be transformed to

$$\text{a)}\quad N\frac{\hbar c}{\lambda} = \frac{E_{\mathrm{exc}}(N)}{2\pi},$$

$$\text{b)}\quad \lambda = \frac{\hbar c}{E^*(N)},$$

$$\text{c)}\quad E^*(N) = \frac{E_{\mathrm{exc}}(N)}{N\,2\pi} = \frac{E_{\mathrm{BB}}}{N\,2\pi}. \tag{8.93}$$

Consequently, for the cases $N = 3, 6, 12$, Eq. (8.93 c) yields

$$\text{a)}\quad E^*(3) = E_{\mathrm{BB}}/6\pi = 5.060 \cdot 10^{89}\ \mathrm{eV},$$

$$\text{b)}\quad E^*(6) = E_{\mathrm{BB}}/12\pi = 2.530 \cdot 10^{89}\ \mathrm{eV},$$

$$\text{c)}\quad E^*(12) = E_{\mathrm{BB}}/24\pi = 1.265 \cdot 10^{89}\ \mathrm{eV}. \tag{8.94}$$

The energies (8.94 a) and (8.94 b) are in excellent agreement with dark energy $E_{\mathrm{d}} = 5.017 \cdot 10^{89}$ eV (see Eq. (8.10)) of the sterile neutrinos and with the half dark energy $\frac{1}{2} E_{\mathrm{d}} = 2.509 \cdot 10^{89}$ eV (see Eqs. (8.11) and (8.12)) of the decay products of the sterile neutrinos. To the energy (8.94 c), we will return in Eqs.

(8.115) and (8.116). Using the results (8.93 b) and (8.94), we obtain the wavelength λ of the zero-point oscillations to

$$\text{a)}\quad \lambda = \frac{\hbar c}{E^*(3)} = \frac{\hbar c}{E_{\mathrm{BB}}/6\pi} = 3.90\cdot 10^{-95}\ \mathrm{cm}\,,$$

$$\text{b)}\quad \lambda = \frac{\hbar c}{E^*(6)} = \frac{\hbar c}{E_{\mathrm{BB}}/12\pi} = 7.80\cdot 10^{-95}\ \mathrm{cm}\,,$$

$$\text{c)}\quad \lambda = \frac{\hbar c}{E^*(12)} = \frac{\hbar c}{E_{\mathrm{BB}}/24\pi} = 1.56\cdot 10^{-94}\ \mathrm{cm}\,. \tag{8.95}$$

Introducing the reduced wavelength $\lambda\!\!\!^{-} = \lambda/2\pi$, Eq. (8.95) provides

$$\text{a)}\quad \lambda\!\!\!^{-} = \frac{\hbar c}{2\pi E^*(3)} = \frac{\hbar c}{E_{\mathrm{BB}}/3} = 6.21\cdot 10^{-96}\ \mathrm{cm}\,,$$

$$\text{b)}\quad \lambda\!\!\!^{-} = \frac{\hbar c}{2\pi E^*(6)} = \frac{\hbar c}{E_{\mathrm{BB}}/6} = 1.24\cdot 10^{-95}\ \mathrm{cm}\,,$$

$$\text{c)}\quad \lambda\!\!\!^{-} = \frac{\hbar c}{2\pi E^*(12)} = \frac{\hbar c}{E_{\mathrm{BB}}/12} = 2.48\cdot 10^{-95}\ \mathrm{cm}\,. \tag{8.96}$$

By generalization of Eq. (8.96), we find $R_{\mathrm{BB}} = \lambda\!\!\!^{-}/N = \hbar c/E_{\mathrm{BB}}$, i.e. we have $\lambda\!\!\!^{-} = NR_{\mathrm{BB}}$. Therefore, we can assume that the reduced wavelengths (8.96) lie in the wavelength range $\lambda\!\!\!^{-}_{\mathrm{BB}} = R_{\mathrm{BB}} < \lambda\!\!\!^{-} < \lambda\!\!\!^{-}_{\hat{\nu}} = R_{\hat{\nu}}$ of the massless universe, where the distance $R_{\hat{\nu}} = 4.938\cdot 10^{-68}\mathrm{cm}$ is defined by Eq. (8.32). Thus, for $\lambda\!\!\!^{-}_{\hat{\nu}}$, we have the condition $\lambda\!\!\!^{-}_{\hat{\nu}} = NR_{\mathrm{BB}} = R_{\hat{\nu}}$, whereat the number N of the sterile neutrino-antineutrino pairs is determined by $N = R_{\hat{\nu}}/R_{\mathrm{BB}} = 2.387\cdot 10^{28}$. Then, the wavelength $\lambda\!\!\!^{-}_{\mathrm{Pl}} = NR_{\mathrm{BB}} = R_{\mathrm{Pl}}$ is also given by $N = R_{\mathrm{Pl}}/R_{\mathrm{BB}} = 7.812\cdot 10^{62}$.

With that, we have shown that the massless universe and anti-universe are connected with each other by the zero-point oscillations as a result of the Dirac fermion-antifermion theory.

To possible zero-point oscillations of the present (massive) universes, we will return in Sec. 8.5.

Using the data of Tables III to V, the corresponding results, derived in this Sec. 8.4 for the massless universe, are also valid for the massless anti-universe.

8.5 The particle horizon

According to Weinberg [7], the particle horizon distance $d_{\max}(t) = d_{\max}(z)$ of the massive universe (see Sec. 3.2.4) is defined by

$$d_{\max}(t) = R(t)\int_0^t \frac{c\,dt'}{R(t')}\,. \tag{8.97}$$

Using Eqs. (2.5) and (2.6), for the present (massive) universe ($z = 0$), by its age $t = t_0$ (see Tables I and V), the value $d_{\max}(t_0) = d_{\max}(0)$ is given by

$$d_{\max}(0) = \frac{c}{H_0}\int_0^1 \frac{dx}{x^2\,(\Omega_\Lambda + \Omega_{\mathrm{m}} x^{-3} + \Omega_{\mathrm{r}} x^{-4})^{1/2}}\,, \tag{8.98}$$

where $x \equiv R/R_0 = 1/(1+z)$ (see Eq. (3.59)) and the corresponding quantities are explained by Tables I and V. Using $\tilde{x} \equiv \tilde{R}/\tilde{R}_0 = 1/(1+z)$ (see Eq. (3.58)), we can introduce

$$\tilde{d}_{\max}(0) = \frac{c}{H_0 \times (1+z_{\mathrm{M}})}\int_0^1 \frac{d\tilde{x}}{\tilde{x}^2\,(\Omega_\Lambda + \Omega_{\mathrm{m}} \tilde{x}^{-3} + \Omega_{\mathrm{r}} \tilde{x}^{-4})^{1/2}}\,. \tag{8.99}$$

However, because of the new inflation model (see Sec. 3.2.4), Eq. (8.99) yields again the result (8.98), so that it is not necessary to discuss the variant (8.99).

Therefore, we consider only the general expression $d_{\max}(t) = d_{\max}(z)$ of the present universe, for which we find

$$d_{\max}(z) = \frac{c}{H_0}\,\frac{1}{1+z}\int_0^{1/(1+z)} \frac{dx}{x^2\,(\Omega_\Lambda + \Omega_{\mathrm{m}} x^{-3} + \Omega_{\mathrm{r}} x^{-4})^{1/2}}\,. \tag{8.100}$$

Using, for example, $R \propto t^{1/2}$ (radiation(r)-dominated universe [1-4]) or $R \propto t^{2/3}$ (matter(m)-dominated universe [1-4]), these particle horizon distances $d_{\max}(t)$ can be defined also by $d_{\max}(t) = 2ct$ or $d(t) = 3ct$ because we know correspondingly the time $t = t(z)$ (see Eqs. (2.19), (7.9) and (7.11)).

Because of the lower integration limit zero, Eq. (8.100) must be transferable to the joint origin of the present universes (see also Eqs. (8.103) to (8.105)) via the assumption of zero-point oscillations (see below), i.e. for $dx > 0$ at the total universe ($R_{\mathrm{BB}} \le R \le R_0$) by Eq. (8.100) we can expect the condition

$$0 \le d_{\max}(z) \le d_{\max}(0)\,, \tag{8.101}$$

whereas at $dx < 0$ for total anti-universe ($-R_0 \le -R \le -R_{BB}$) by Eq. (8.100) we can assume the condition

$$-d_{max}(0) \le -d_{max}(z) \le 0 . \tag{8.102}$$

However, for $x = 1/(1+z) \to 0$, the integral (8.100) of the particle horizon distance $d_{max}(z)$ would be infinite. Consequently, we must determine a lower integration limit. For this goal, assuming as the lower integration limit $x = x_{BB} = 1/(1+z_{BB}) = R_{BB}/R_0 = 1.505 \cdot 10^{-124}$, we can convert Eq. (8.100) into

$$d_{max}(z) = \frac{c}{H_0} \frac{1}{1+z} \int_{1/(1+Z_{BB})}^{1/(1+z)} \frac{dx}{(\Omega_\Lambda x^4 + \Omega_m x + \Omega_r)^{1/2}} . \tag{8.103}$$

For $1/(1+z) \to 1/(1+z_{BB}) = 1.505 \cdot 10^{-124}$, Eq. (8.103) provides

$$d_{max}(z) = \frac{c}{H_0} \frac{1}{1+z} \int_{1/(1+Z_{BB})}^{1/(1+z)} \frac{dx}{\Omega_r^{1/2}} . \tag{8.104}$$

Using the above-mentioned transition to the joint origin of the present universes, we must apply $\Omega_r = \frac{1}{2} N(T)\, \Omega_\gamma$ (see Eq. (2.8)) and $N(T) = 1/2\, \Omega_\gamma$ of the quantum gravity (see Sec. 6), so that Eq. (8.104) yields

$$d_{max}(z) = \frac{c}{H_0} \frac{1}{1+z} \left[\frac{2}{1+z} - \frac{2}{1+z_{BB}} \right] . \tag{8.105}$$

Indeed, we obtain $d_{max}(z_{BB}) = 0$. This result is plausible because, for example, at $R = R_{BB}$ by the big bang the origin of the present universe begins.

Now, we prove the zero-point oscillations for the present universes ($z = 0$). For this goal, using again $N(T) = 1/2\, \Omega_\gamma$ of the quantum gravity (see above), the mass density $\rho_r = \frac{1}{2} N(T)\, \Omega_\gamma \rho_{0C} (1+z)^4$ (see Eqs. (3.17) and (3.50)) is converted into the present, quantum-theoretical density

$$\rho_r = \tfrac{1}{4} \rho_{0C} = \tfrac{1}{4} \frac{3 H_0^2}{8\pi G_N} , \tag{8.106}$$

which in contrast to Eq. (3.50) is also valid totally for the present universes because of the zero-point oscillations. Then, using Eq. (2.5), we find

$$\frac{\dot{R}}{R} = \tfrac{1}{2} H_0 . \tag{8.107}$$

Thus, via the zero-point oscillations (see Sec. 8.4), we can assume the wavelength $\lambda\!\!\!^{-}_0 = \hbar c/(E_{BB}/N) = NR_{BB} = R_0$ (with $N = R_0/R_{BB} = 6.646 \cdot 10^{123}$) as well as the velocity $\dot{R} = c$, so that by Eq. (3.41) the result (8.107) yields

$$R = 2\frac{c}{H_0} = 2R_0 = 2NR_{BB}, \tag{8.108}$$

where $R = 2R_0$ describes the distance between the present universe an anti-universe as a result of the zero-point oscillations. Consequently, between the lower integration limit (see Eqs. (8.103) to (8.105)) and the total particle number N we get the connection

$$\frac{1}{N} = \frac{R_{BB}}{R_0} = x = x_{BB} = \frac{1}{1+z_{BB}} = 1.505 \cdot 10^{-124}. \tag{8.109}$$

With that, we have supported the assumption that the present universe and anti-universe can be attributed also to zero-point oscillations.

Using the data of Tables III to V, the results, derived in this Sec. 8.6 for the present universe, are also valid for the present anti-universe.

Eqs. (8.101) and (8.102) describe two opposite directions, in which the present universe as well as the present anti-universe (see Table IV) expand simultaneously. This result is supported, for example, by the Dirac fermion-antifermion theory and the simultaneous observation of particles and antiparticles by nuclear reactions as well as the CPT symmetry.

With that, we have proved the simultaneous separation of matter and antimatter.

The total energy of the present universe ($z = 0$) can be evaluated by the present critical energy density $\rho_{0C}\, c^2 = 4.77 \cdot 10^3\ \mathrm{eV\ cm^{-3}}$ (see Table I) and the distance $R_0 = 1.375 \cdot 10^{28}\,\mathrm{cm}$ (see Eqs. (3.41) and (3.59)) to

$$E = \rho_{0C}\, c^2\, \tfrac{4}{3}\pi R_0^3 = 5.194 \cdot 10^{88}\ \mathrm{eV}. \tag{8.110}$$

Consequently, the total energy $2E$ of the present universe and anti-universe is given by

$$2E = 2\rho_{0C}\, c^2\, \tfrac{4}{3}\pi R_0^3 = 1.039 \cdot 10^{89}\ \mathrm{eV}. \tag{8.111}$$

This energy (8.111) can be determined also by the smallest thermal (particle) energy $kT_{BB} = 1.536 \cdot 10^{-35}\,\mathrm{eV}$ (see Eq. (8.27)), multiplied by the neutrino-

antineutrino pair number $N = R_0/R_{BB} = 6.646 \cdot 10^{123}$ (see Sec. 8.4). Then the total energy of the present universe and anti-universe is also given by

$$2E = N \times kT_{BB} = 1.038 \cdot 10^{89}\ \text{eV}. \tag{8.112}$$

This excellent agreement supports the assumption that the present universe and anti-universe can be described by zero-point oscillations.

8.6 The hypothetical superforce of the particle interactions

Using in the massless universe the Planck energy $E_{Pl} = (\hbar c^5/G_N)^{1/2}$ as mean energy (see, e.g., Eqs. (8.80) to (8.83)) for the big bang at $R = R_{BB}$ (see Eq. (8.23)), we obtain the lowest, mean energy $\overline{E}_{BB}$ of the big bang by the gravitational potential energy to

$$\overline{E}_{BB} = \frac{G_N}{c^4}\frac{E_{Pl} \times E_{Pl}}{R_{BB}} = \frac{\hbar c}{R_{BB}} = 9.537 \cdot 10^{90}\ \text{eV}. \tag{8.113}$$

The result (8.113) is equivalent to the energy (8.28). This energy (8.28) was interpreted as the potential energy $E_{BB} = \hbar c/R_{BB}$ of a new attractive force, which is now assumed as the attractive, hypothetical superforce of the particle interactions.

Therefore, we can assume that the lowest mean energy (8.113) of the big bang corresponds to the highest value of the potential energy $E = \hbar c/R$ of the attractive, hypothetical superforce of the particle interactions.

At $R = R_{Pl} = (\hbar G_N/c^3)^{1/2}$, this hypothetical superforce has its lowest value E_{Pl}, where the unification of the strong and electroweak interaction (grand unification) begins because the gravitational force and this hypothetical superforce are here again equivalent because of

$$E_{Pl} = \frac{G_N}{c^4}\frac{E_{Pl} \times E_{Pl}}{R_{Pl}} = \frac{\hbar c}{R_{Pl}} = 1.220932 \cdot 10^{28}\ \text{eV}. \tag{8.114}$$

Because of Eq. (8.113), the lowest mean gravitational energy $\overline{E}_{BB} = \hbar c/R_{BB}$ of the big bang and the highest energy value $E_{BB} = \hbar c/R_{BB}$ (see above) of this hypothetical superforce are conserved quantities with the same values.

Consequently, the greatest possible value $2\overline{E}_{\max}$ of the mean, gravitational potential energy of the massless universe and anti-universe (distance $2R_{\mathrm{BB}}$) is found by the energy $E_{\mathrm{BB}}/24\pi = 1.265 \cdot 10^{89}$ eV (see Eq. (8.94 c)) to

$$2\overline{E}_{\max} = \frac{G_N}{c^4} \frac{(E_{\mathrm{BB}}/24\pi) \times (E_{\mathrm{BB}}/24\pi)}{2R_{\mathrm{BB}}}, \tag{8.115}$$

so that for the massless universe alone we obtain the greatest possible mean, gravitational potential energy $\overline{E}_{\max}$ to

$$\overline{E}_{\max} = \cdot \frac{G_N}{c^4} \frac{(E_{\mathrm{BB}}/24\pi) \times (E_{\mathrm{BB}}/24\pi)}{2 \times 2R_{\mathrm{BB}}} \cong 2.560 \cdot 10^{212}\ \mathrm{eV} \tag{8.116}$$

in accordance with the mean energies (8.82) and (8.87), i.e. the energy conservation is guaranteed.

More generally, for the potential $V(R)$ of the hypothetical superforce of the particle interactions in the massless universe, we can write

$$V(R) = -\frac{\hbar c}{R}, \tag{8.117}$$

so that we find the law of this hypothetical superforce to

$$\vec{K}(\vec{R}) = -\frac{\partial V(R)}{\partial \vec{R}} = -\frac{\vec{R}}{R} \frac{dV(R)}{dR} =$$

$$= -\frac{\hbar c}{R^2} \frac{\vec{R}}{R} \quad \text{for} \quad R_{\mathrm{BB}} \le R \le R_{\mathrm{Pl}}. \tag{8.118}$$

Using the data of Tables III and IV, the results, derived in this Sec. 8.6 for the massless universe, are also valid for the massless anti-universe.

9 The explanation of the present, dark matter and dark energy

According to Table I, the present, (ionizable or baryonic) matter is $\Omega_{\mathrm{b}} = 0.0499$, whereas the present, dark matter Ω_{dm} and dark energy Ω_{Λ} are given by $\Omega_{\mathrm{dm}} = 0.265$ and $\Omega_{\Lambda} = 0.685$.

They possess the total relation $\Omega_{\mathrm{b}} + \Omega_{\mathrm{dm}} + \Omega_{\Lambda} = 1$.

In Sec. 5.1, we have shown that the joint origin of the present, dark matter and dark energy [1-4] is based on the three sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$. Because of the decay of these sterile neutrinos via the gravitation in one photon (γ) with the energy of their half rest energy and one sterile, massive neutrino relic ($\hat{\nu}_{1/2}$) with the energy of their half rest energy, we assume that these two decay products can be unified also as their components to the sterile neutrinos. Therefore, we can together consider the present, dark matter and dark energy, i.e. the sum $\Omega_{\mathrm{dm}} + \Omega_\Lambda = 0.265 + 0.685 = 0.950$.

The decay of these sterile neutrinos is again determined by the universal decay law $N(t) = N_0\, \mathrm{e}^{-t/\pi_{\hat{\nu}}}$, where $\tau_{\hat{\nu}}$ is the lifetime $\tau_{\hat{\nu}} = 35.11\,\mathrm{Gyr}$ of the sterile neutrinos (see Eq. (8.15)). Assuming $N_0 = \Omega_{\mathrm{dm}} + \Omega_\Lambda$, the remaining dark matter and dark energy is given by $N(t) = (\Omega_{\mathrm{dm}} + \Omega_\Lambda)\, \mathrm{e}^{-t/\pi_{\hat{\nu}}}$. Using the Hubble (H) time $\tau_{\mathrm{H}} = 1/H_0 = 14.53\,\mathrm{Gyr}$ (see Table I), which is valid for all galaxies (which expand by the velocity $v = H_0 R$ at very small redshifts), we can assume the time $t = \tau_{\hat{\nu}} - \tau_{\mathrm{H}} = 20.58\,\mathrm{Gyr}$, i.e. $\mathrm{e}^{-(\tau_{\hat{\nu}} - \tau_{\mathrm{H}})/\tau_{\hat{\nu}}} = 0.5565$. This assumption is supported by the fact that this Hubble time defines also (according to Eq. (3.41)) the scale factor $R_0 = c\tau_{\mathrm{H}} = c/H_0$ of the present (massive) universe for $z = 0$ by Eq. (3.59).

Then, the decay products of the dark matter and dark energy are defined via the slow decay process

$$(\Omega_{\mathrm{dm}} + \Omega_\Lambda)(1 - \mathrm{e}^{-(\tau_{\hat{\nu}} - \tau_{\mathrm{H}})/\tau_{\hat{\nu}}}) = 0.1175 + 0.3038 = 0.4213 \tag{9.1}$$

to

$$\{\Omega_{\mathrm{dm}}(\gamma) + \Omega_\Lambda(\gamma)\}_{\mathrm{decay}} = 0.0588 + 0.1519 = 0.2107 \tag{9.2}$$

and

$$\{\Omega_{\mathrm{dm}}(\hat{\nu}_{1/2}) + \Omega_\Lambda(\hat{\nu}_{1/2})\}_{\mathrm{decay}} = 0.0588 + 0.1519 = 0.2107\,. \tag{9.3}$$

The dark radiation energy (9.2) lies in an invisible range of the radiation spectrum, whereas dark matter (9.3) is invisible because it is neutral.

Because of Eqs. (9.1) to (9.3), the remaining dark matter and dark energy (see above) is determined to

$$(\Omega_{\mathrm{dm}} + \Omega_\Lambda)\, \mathrm{e}^{-(\tau_{\hat{\nu}} - \tau_{\mathrm{H}})/\tau_{\hat{\nu}}} = 0.1475 + 0.3812 = 0.5287\,. \tag{9.4}$$

However, after the decay products (9.1) to (9.3), the strong repulsion (rep) of the dark energy is large enough, so that momently it leads to the breakup of the sterile neutrinos of the dark matter and dark energy in their components, i.e. from Eq. (9.4) we get

$$\{\Omega_{\mathrm{dm}}(\gamma)+\Omega_{\Lambda}(\gamma)\}_{\mathrm{rep}} = 0.0738+0.1906 = 0.2644 \tag{9.5}$$

and

$$\{\Omega_{\mathrm{dm}}(\hat{\nu}_{1/2})+\Omega_{\Lambda}(\hat{\nu}_{1/2})\}_{\mathrm{rep}} = 0.0738+0.1906 = 0.2644\,. \tag{9.6}$$

We interpret the result (9.5) as dark radiation energy $\{\Omega_{\mathrm{dm}}(\gamma)+\Omega_{\Lambda}(\gamma)\}_{\mathrm{rep}}$, which lies in the invisible range of the radiation spectrum, whereas the expression (9.6) is interpreted as dark matter $\{\Omega_{\mathrm{dm}}(\hat{\nu}_{1/2})+\Omega_{\Lambda}(\hat{\nu}_{1/2})\}_{\mathrm{rep}}$ because it is neutral.

These dark radiation energy $\{\Omega_{\mathrm{dm}}(\gamma)+\Omega_{\Lambda}(\gamma)\}_{\mathrm{rep}}$ and dark matter $\{\Omega_{\mathrm{dm}}(\hat{\nu}_{1/2})+\Omega_{\Lambda}(\hat{\nu}_{1/2})\}_{\mathrm{rep}}$ can be considered independent on the corresponding decay products (9.2) and (9.3), since the expressions (9.5) and (9.6) are formed momently in direct neighbourhood of the (ionisable) matter Ω_{b}, whereas by the slow decay process the decay products (9.2) and (9.3) of the sterile neutrinos are distant from the (ionisable) matter Ω_{b}.

In this connection, for the slow decay process (9.1) to (9.3), we must consider that the photons (9.2) move with the velocity $v = c$, whereas the massive, sterile neutrino relics (9.3) possess the velocity $v \approx c$, so that this very small velocity difference has no influence on their distributions in the gigantic, present universe.

A completely other situation exists for the results (9.4) to (9.6), where by the momentary breakup of the sterile neutrinos in their components the photon packet (9.5) with the velocity $v = c$ and the massive, sterile neutrino relic packet (9.6) with the still relativistic velocity $v << c$ are formed, so that by the gravitational attraction the massive, sterile neutrino relic packet (9.6) is connected with the (ionisable) matter Ω_{b}, whereas the photon packet (9.5) escapes from the (ionisable) matter Ω_{b} in the gigantic, present universe.

Then, the total matter is $\Omega_b + \left\{\Omega_{dm}(\hat{\nu}_{1/2}) + \Omega_\Lambda(\hat{\nu}_{1/2})\right\}_{rep} = 0.3143$, so that we obtain the ratio

$$\frac{\text{total matter}}{\text{matter}} = \frac{\Omega_b + \left\{\Omega_{dm}(\hat{\nu}_{1/2}) + \Omega_\Lambda(\hat{\nu}_{1/2})\right\}_{rep}}{\Omega_b} = 6.30\,, \tag{9.7}$$

which agrees excellently with the experimental value $(\Omega_b + \Omega_{dm})/\Omega_b = 6.31$. This agreement confirms our above-mentioned assumptions.

After the processes (9.4) to (9.6), because of the particle conservation, the results (9.1) to (9.6) can be summarized in the gigantic, present universe, so that we get as dark matter

$$\Omega_{dm} = \left\{\Omega_{dm}(\hat{\nu}_{1/2}) + \Omega_\Lambda(\hat{\nu}_{1/2})\right\}_{rep} = 0.2644 \tag{9.8}$$

and as dark energy

$$\begin{aligned}\Omega_\Lambda &= \left\{\Omega_{dm}(\gamma) + \Omega_\Lambda(\gamma)\right\}_{rep} + \left\{\Omega_{dm}(\gamma) + \Omega_\Lambda(\gamma)\right\}_{decay} + \\ &+ \left\{\Omega_{dm}(\hat{\nu}_{1/2}) + \Omega_\Lambda(\hat{\nu}_{1/2})\right\}_{decay} = 0.2644 + 0.2107 + 0.2107 = 0.686\end{aligned} \tag{9.9}$$

with $\Omega_{dm} + \Omega_\Lambda = 0.950$, i.e. the energy conservation is guaranteed.

With that, we have found a simple model for the explanation of the dark matter and dark energy, i.e. it was proved that the breakup as well as decay products of the sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{dm}$ and $\hat{\nu}_b$ form the dark matter and the dark energy as their joint origin.

Using the data of Tables III to V, this model, derived in this Sec. 9 for the universe, is also valid for the anti-universe.

10 Summary

We have shown that together with the new inflation model the Friedmann-Lemaitre Equations describe excellently the massive, Euclidian universe and anti-universe, whereat we have used particle-defined and antiparticle-defined, (present), cosmological parameter values. However, the old inflation model, which can indeed explain qualitatively the pressing problems within the cosmology, cannot be understood in the frame work of the known particle

physics. This difficulty does not exist for the new inflation model, which is completely described by the SUSY GUT particles (X and Y gauge bosons as well as magnetic monopoles).

Using the results of the neutrino oscillation experiments together with cosmological data, we have presented an approximate solution for the neutrino problem (by the estimation of the neutrino masses and the neutrino densities of the universe). Via this estimation and the magnetic neutrino moment of the standard $SU(2)\times U(1)$ electroweak theory, we have derived the masses of the X and Y gauge bosons. We have also obtained the mass of the magnetic monopoles via the big bang model, so that in accordance with the particle physics we have exactly determined the coupling constant of the supersymmetric grand unified theories which characterize the early universe. We have so found reliable arguments for the existence of the heavy and the sterile neutrinos.

Using the two scale factors of absorption at the redshift condition before and after the above-mentioned inflation as different functions of the Hubble constant, we have obtained an exact and unique description of this inflation and its transition to the standard model, so that we have confirmed the presented, qualitative solutions for the pressing problems of the standard cosmology as the horizon problem, the monopole problem and the initial condition problem as well as the flatness problem.

Via the matter-dominated universe, we have shown that the influence of the dark energy, which determines the present-day accelerated expansion of the universe, ends in equilibrium with a negative acceleration, whereat we have explained the Pioneer anomaly via this mean negative acceleration as well as the astronomical unit changing. After the end of the present, accelerated expansion, i.e. in future, this present exponential expansion gives way to a slow linear expansion of the universe by the astronomical unit changing. Therefore, the age of the universe for its final state, which is determined by the redshift condition of the electron neutrino, can be estimated to $6883\,\mathrm{Gyr}$ via the transition from the final state of the massive universes to the big bang as the beginning of the massless universes. Via the neutrinos, we have obtained the redshift values for the beginning, the half and the end of the reioinization.

We have proved that from the big bang ($R = \pm R_{BB}$) by the momentary formation of universe and anti-universe, which expand in opposite directions because of their momentum with opposite sign, the virtual particle-antiparticle (photon-photon) pairs of the quantum vacuum are separated into real radiation and particle fields of the massless universe ($R_{BB} \leq R \leq R_{Pl}$) as well as real radiation and antiparticle fields of the massless anti-universe ($-R_{Pl} \leq -R \leq -R_{BB}$). These massless universes were described via the gravitation.

On this way, we have explained generally the long-sought quantum gravity as a result of the unique connection between vacuum energy density and cosmological "constant", i.e. we have found a simple solution for this four-dimensional quantum gravity, which influences not only the massless but also the massive universes because their space-time continuum is penetrated with the quantum vacuum.

Generally, this four-dimensional quantum gravity explains the particle formation, which permits the conclusion that at $R = \pm R_{Pl}$ all particles and antiparticles have the Planck energy as relativistic energy in the massless and massive universe as well as anti-universe.

The excellent usefulness of this four-dimensional quantum gravity is shown at its application for the transition from the final state of the massive universes to the big bang as the beginning of the massless universes as a consequence of the dark energy, which is completely converted into massless photons and the corresponding, massive relics by the decay of the sterile neutrinos via the gravitation. This result supports generally our hypothesis from a joint origin of the dark matter and the dark energy by sterile neutrinos.

By the transition from the final state of the universe to the big bang, we have clearly shown that the evolution of the total (massless and massive) universe is cyclic. In contrast to the present theory, which postulates an infinite, Euclidian universe, we have proved that the Euclidian universe is finite.

We have reliably estimated the data for the big bang, which takes place at the distance $R = R_{BB} = 2.069 \cdot 10^{-98}$ m and the time $t = t_{BB} = 6.901 \cdot 10^{-107}$ s. Its vacuum energy density or cosmological "constant" are defined by

$\rho_{\text{vac}}(R_{\text{BB}})c^2 = 4.227 \cdot 10^{247}\,\text{eV}\,\text{cm}^{-3}$ or $\Lambda = 1.406 \cdot 10^{192}\,\text{m}^{-2}$. The lifetime $\tau_{\hat{\nu}}$ of the sterile neutrinos was found to $\tau_{\hat{\nu}} = 35.11\,\text{Gyr}$.

We have demonstrated that in the final state of the massive universes the vacuum energy density (determined by the dark energy) yields a gigantic energy increase in consequence of the heat accumulation by the decay products of the sterile neutrinos via the gravitation.

On this way, via the total, mean energy of the universes (anti-universes), we have proven uniquely that the energy conservation is fulfilled between the massless and massive universe (anti-universe), whereat the basis of the total, mean energies for these universes (anti-universes) is the Planck energy as greatest possible, relativistic particle energy.

This gigantic energy increase leads to a heat accumulation, which produces an overheat of the final state of the massive universes. This overheat means the heat death of these massive universes and simultaneously the birth of the massless universes by a hot big bang because of the particle formation in the quantum gravity (energy conservation).

We have uniquely shown that the origin of the massless universe and anti-universe as well as the present universe and anti-universe results from zero-point oscillations. We have reasonably proved the separation of matter and antimatter, which is realized by the simultaneous, opposite expansions of the total universe and anti-universe.

We have explained the attractive, hypothetical superforce of the particle interactions. We have found a reasonable explanation for the present, dark matter and dark energy.

Thus, using the data of Tables III to V, we have realized that all results, derived in this work for the universe, are also valid for the anti-universe.

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